

Network Connectivity and Systematic Risk

Monica Billio* Massimiliano Caporin[†] Roberto Panzica[‡]
Loriana Pelizzon[§]

January 2015

PRELIMINARY AND INCOMPLETE. PLEASE DO NOT QUOTE

Abstract

The need for understanding the propagation mechanisms behind the recent financial crises lead the increased interest for works associated with systemic risks. In this framework, network-based methods have been used to infer from data the linkages between institutions (or companies). Part of the literature postulates that systemic risk is strictly related (if not equal) to systematic risk. In this work, we elaborate on this hypothesis and introduce a modelling framework where systemic and systematic risks co-exist. The model is a variation of the traditional CAPM/APT model where networks are used to infer the exogenous and contemporaneous links across assets. The systematic risk component acts in an additive way on both the systematic and idiosyncratic risk components. Our proposed methodology is verified both on simulations as well as on real data.

Keywords: CAPM, Volatility, Network, interconnections, systematic risk, systemic risk.

JEL Classification: G10, G12, F35, C58.

*University Ca' Foscari Venezia (Italy)

[†]University of Padova (Italy)

[‡]SAFE-Goethe University Frankfurt (Germany)

[§]SAFE-Goethe University Frankfurt (Germany)

1 Introduction

The term “Systematic risk” is a well established concept that derives from the seminal work on portfolio choice proposed by Markowitz (1952) and extended in a general equilibrium framework by Sharpe (1964), Lintner (1965a,b), and Mossin (1966) and in the Arbitrage Price Theory model by Ross (1976). It refers to the risk an investor of a well-diversified portfolio is exposed to, which stems from the dependence of returns to common factors.

On the other side, the definition of ”Systemic risk” is not well defined throughout the literature and, as a result, can be measured from a wide range of perspectives.

According to Acharya and Yorulmazer (2002), Nier et al. (2007) and De Bandt et al. (2010) systemic risk materialises through (1) “pure” contagion, (2) exposure to common factors, (3) herding behaviour causing informational contagion, and (4) feedback effects from endogenous fire sales. Hartmann (2002) argues systemic risk stems from either build-up imbalances, contagion or large shocks.

The broad definition provided above links contagion risk to systemic risk as well as exposure to common factors, that in principle is largely related to systematic risk. A natural statistical model for capturing systemic risk exposure due to linkages between institutions is a network model, which is commonly used to describe features of a network of connections.

In this paper we provide a unique framework for systematic risk and network connections and estimate the feedbacks among network exposures and common factors and the impact of them on the risk exposures and risk premia of stock returns. More specifically, we look to the the interactions of the four ways through which a broad definition of systemic risk materialize, i.e. the relationships between (i) “pure” contagion, (ii) herding behaviour causing informational contagion, and (iii) feedback effects from endogenous fire sales that could be well captured by a network model and exposures to common factors that could be considered *per se* as systematic risk exposure.

A growing literature investigates the role of interconnections between different firms

and sectors, functioning as a potential propagation mechanism of idiosyncratic shocks throughout the economy. Acemoglu et al. (2011) use network structure to show the possibility that aggregate fluctuations may originate from microeconomic shocks to firms; Billio, Gray, Getmansky, Lo, Merton and Pelizzon (2014) use contingent claim analysis and network measures to highlight interconnections among sovereign, banks and insurances. There are several other contribution in the literature on network analysis: see Billio, Getmansky, Lo, and Pelizzon (2012), Diebold and Yilmaz (2014) and Hautsch, Schaumburg, and Schienle (2012, 2013) and Barigozzi and Brownlees (2014). Network interconnections and the effects called network externalities that arises from small and local shocks that can become big and global is a possibility discarded in standard asset pricing and macro-economics models due to a “diversification argument”. As argued by Lucas (1977), among others, microeconomic shocks would average out and thus, would only have negligible aggregate effects. Similarly, these shocks would have little impact on asset prices. However, there is already a growing literature on the role of sectorial shocks in macro fluctuations; examples include Horvath (1998, 2000), Dupor (1999), Shea (2002), and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2011).

The need for understanding the propagation mechanisms behind the recent financial crises leads to an increased interest for works associated with systemic risks. In this framework, network-based methods described above will be used to infer from data the linkages between institutions (or companies). Part of the literature postulates that systemic risk is strictly related (if not equal) to systematic risk and therefore there is no need to distinguish among the two. With this paper instead we argue that it is important to disentangle the channels through which risk propagates: spillover or contagion channels versus exposures to common factors.

In fact, the contribution of this paper to this literature is to propose a modelling framework where network interconnections and common factors risks co-exist. The proposed model is a variation of the traditional CAPM/APT model where networks are used to infer the exogenous/lagged and contemporaneous links across assets. This approach

allows us to decompose the risk of a single assets (or a portfolio) in four components: the two classical systematic and idiosyncratic components and (i) the impact of the asset interconnections on the systematic risk component, that is the contribution of network exposure to the systematic risk component and (ii) the effect of interconnections on the idiosyncratic risk on the systematic risk component, that is the amplification of idiosyncratic risks that generates systematic/non diversifiable risk. Our approach allows us also to decompose the risk premium component of returns in three components: the risk premium associated with (i) common factors exposures, (ii) impact of asset connections to common factors, and (iii) the amplification effects of idiosyncratic risk.

Our proposed methodology is verified both on simulations as well as on real data. The simulation analysis has been provided mostly for clarifying the decomposition of the contribution to volatility of a single asset or a portfolio of network interconnections. In the empirical analysis, we use the Granger causality approach proposed by Billio et al. (2012) to extract connections among institutions from the Fama-French sectors portfolio and the approach used in spatial econometrics, namely the use of concentrated likelihoods, to estimate the exposures of returns to network connections and therefore their contribution to sectors volatility and beta exposures.

The remainder of the paper is organized as follows. Section 2 describes network models. Section 3 presents the extension to the CAPM/APT model to interconnectness exposures. Section 4 presents the estimation methodology. Section 5 shows the results of the empirical analysis on the Fama-French sector portfolios. Section 6 concludes.

2 Network Models in Finance

Network models have seen an extremely diverse array of applications: in the social sciences with studies related to social networking on websites such as Facebook, in the natural sciences with application to protein interactions, in government intelligence where they are used to analyse terrorist networks, in politics with application to bill co-authorship,

in economics with potential used in labour markets analysis, and many other areas. In finance, network models have most frequently been used to assess financial stability. In fact, interconnections among financial institutions create potential channels for contagion and amplification of shocks to the financial system that can be also propagated to the “real economy”.

Applications in this area have gauged considerable interest in the aftermath of the 2007-2009 financial crisis. Network representation of interconnections ranges from linkages extracted from balance-sheet information to connections estimated by means of econometric approaches from either market data, accounting data or macroeconomic data.

The majority of such “real-world” networks have been shown to display structural properties that are neither those of a random graph, nor those of regular lattices.

In order to evaluate the relevance and the price of interconnections in the financial system it is fundamental to understand all of the channels by which small and local shocks can become big and global.

Empirical network modelling has been conducted for assessing asset pricing linkages via contagion (Allen and Gale 2000; Dasgupta 2004; Leitner 2005, Billio, Getmansky, Lo, and Pelizzon (2012), Diebold and Yilmaz (2013) and Hautsch, Schaumburg, and Schienle (2012, 2013), Brownlees (2014)), linkages via balance sheets (Cifuentes et al 2005; Laguno and Schreft 2001), and how failures of institutions result from mutual claims on each other (Furfine 2003; Upper and Worms 2004; Wells 2004). Babus and Allen (2009) provide a review of network models in finance.

Much of the empirical finance literature has focused on “direct” contagion arising from firms’ contractual obligations. Direct contagion occurs if one firm’s default on its contractual obligations triggers distress (such as insolvency) at a counterparty firm. Researchers’ simulations using actual interbank loan data suggest that “domino defaults” arising from contractual violations are very unlikely, (see Furfine (2003) Eisinger et al. (2006), Upper and Worms (2004); Mistrulli (2007); Degryse and Nguyen (2007), Van Lelyveld and Liedorp (2006) and Alves et al (2013)) though they can be highly destructive in the event

that they do materialise.

Contractual obligations are not the only means by which small and local shocks can spread and generate perverse externalities. Focusing only on direct contagion underestimates the risk of financial crisis given that other important channels exist like common exposures, fire sales, illiquidity spirals and, information spillover. For example, in its survey Upper (2011) reports that simulations using actual interbank loan data suggest that domino defaults are very rare events, and Abbassi, Brownlees, Hans and Podlich (2014) shows that model network structures for a sample of German banks based on CDS data are only marginally explained by direct connections through interbank exposures and common exposures to similar asset classes extracted by accounting data.

The approach that we follow in this paper is that both direct and indirect interconnections extracted from accounting or direct exposures data and market data could co-exist and have implication on the dynamic of the returns of financial assets. Therefore, our approach is very general. We first concentrate on interconnections that could be estimated from market data and then we provide a theoretical extension of the model where also direct linkages like balance-sheet exposures or common exposures to similar asset classes could be included in the framework.

The advantage of using market data to extract linkages has relevant advantages: the data are easily available, have higher frequency (that is more information, and a more up-to-date view of links) and the linkages extracted from market data are forward looking in contrast to balance-sheet/accounting data that provide a pictures of the actual exposures (and might be seen thus as backward-looking). The forward looking interpretation can also supported by the general idea that market prices can be seen as reflecting information available to traders/operators/market participants, and, in equilibrium, correspond to the discounted value of future dividends (thus with a link to fundamental valuations of stocks).

Formally, we could represent networks as nodes that are connected (in general) to a subset of the network total number of nodes, where connections represent links across nodes. A financial system could be represented as a network structure where nodes

represent assets or the value of financial or non financial institutions, and shocks on one asset/institutions are transmitted to the connected ones.

Networks are, in general, graphically represented, and we also provide some examples in the empirical section. Nevertheless, networks have an equivalent (square) matrix representation. Let us call W the K -dimensional square matrix representing a network composed by K financial assets/companies. Each entry $w_{i,j}$ represents the possible connection between assets i and j . A zero entry indicates that the two assets are not connected, while a non-null entry indicates the existence of a connection. Depending on the approach adopted to estimate the network, non-null entries might differ one from the other, that is they track the strength/intensity of the connection, or might be simply equal one to the other, and thus just indicate the existence of a connection. An example of the last case is the following matrix:

$$W = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad (1)$$

where note that the diagonal contains only null elements (each asset is not influencing itself) and the network is not symmetric as the first asset is connected to the fourth one, but the opposite is not true.

Interestingly, matrices similar to that of equation (1) are very common in other economic and statistic applications, those concerning research and studies associated with spatial econometrics and spatial statistics. In these fields, subjects (like towns, buildings, regions) are neighbour one to the other in a physical way, and the W matrices represent the neighbouring relations with entries possibly associated with the physical distance existing between two subjects; they are normally called *spatial matrices*, and are commonly

row-normalized.

Matrix representation of financial networks might thus be seen as the financial parallel of spacial matrices. Clearly, neighbouring relations are no more physical, but are the outcome of a specific model, measuring or estimation approach. Going back to the graphical representation of networks, where nodes are connected one to the other, we might state that connected nodes (assets/firms) are thus neighbour.

Finally, we stress that, if we consider matrices monitoring only the existence of the connection across assets, we adhere to the concept of “first order contiguity ” where a unit entry denotes the existence of a connection and the fact that two assets are neighbour, see LeSage (1999). In addition, by convention in spatial statistic/econometrics, the main diagonal of the W matrix contains zero elements.

In the following, we will clarify how network connections, as monitored by the matrix W will convey relevant information on the evolution of asset returns. In doing that, we do not restrict ourselves to a specific structure of W , that is with a W monitoring the existence of a connection and/or the intensity of the link, but will propose a model which can be used with any form of W . Moreover, according to Elhorst (2003), we will anyway normalize W by row, so that, if we are monitoring only the existence of the connection, we equalize the impact of each unit on all other units.

Later, when moving to the empirical part, we will also briefly discuss alternative methods that can be followed to estimate the existence of a connection across two assets.

3 The systematic effects of network exposure

Since the seminal works of Sharpe (1964), Lintner (1965a,b), and Mossin (1966) linear returns models have attracted a huge interest in the financial economics literature, and have had an extraordinary impact on both research and practice. In the last decades, multifactor generalizations of the CAPM model have been proposed and are now as diffused as the single factor model. The first multifactor models stem from the work of

Ross (1976) on the arbitrage pricing theory, and the most commonly used approaches in pricing take now into account the developments of Fama and French (1993 and 1995), and Carhart (1997), leading to the so-called three-factor and four-factor CAPM models, respectively. The latter approach represents our starting point for the introduction of the impact of network exposure on both the systematic and idiosyncratic risk components. We thus start from a linear model for a K -dimensional set of time t risk asset returns, which we denote by R_t :

$$R_t = \alpha + \beta_M R_t^M + \beta_{HML} R_t^{HML} + \beta_{SMB} R_t^{SMB} + \beta_{MOM} R_t^{MOM} + \varepsilon_t \quad (2)$$

where R_t^M is the returns on the equity market index, R_t^{HML} is the book-to-market factor (see Fama and French, 1995), R_t^{SMB} is the size factor (see Fama and French, 1995), and R_t^{MOM} is the momentum factor (see Carhart, 1997). Moreover, α , β_M , β_{HML} , β_{SMB} and β_{MOM} are K -dimensional vectors of model parameters, and ε_t is the vector of idiosyncratic shocks. The beta vectors monitor the exposure to the common factors and assume a central role in the following analyses. To simplify the following steps, we represent the model in a compact form by collecting the four common factors into a single vector $F_t = [R_t^M \quad R_t^{HML} \quad R_t^{SMB} \quad R_t^{MOM}]'$ and the factor exposure vectors into a matrix $\beta = [\beta_M : \beta_{HML} : \beta_{SMB} : \beta_{MOM}]$.¹ We thus obtain the following representation

$$R_t = \alpha + \beta F_t + \varepsilon_t. \quad (3)$$

Note that, the notation we use, and thus also the following generalizations, can be applied to any collection of risk factors. However, for reasons explained below, the risk factors should not be recovered by means of *statistical* approaches, such as principal component analysis or the estimation of a latent factor model, but must be observed variables.

If we take a pricing perspective, we assume that factors have zero mean, and the model

¹With the symbol $:$ we denote horizontal concatenation of column vectors.

intercept can be replaced by the vector of expected returns

$$R_t = \mathbb{E}[R_t] + \beta F_t + \varepsilon_t. \quad (4)$$

Moreover, expected returns depend on the factor risk premiums Λ obtaining

$$\mathbb{E}[R_t] = r_f + \beta\Lambda. \quad (5)$$

The four-factor CAPM allows decomposing the total risk of the assets into the sum of two components:²

$$\mathbb{V}[R_t] = \beta\Sigma_F\beta' + \Omega_\varepsilon, \quad (6)$$

where $\mathbb{V}[\cdot]$ is the variance operator, $\mathbb{V}[F_t] = \Sigma_F$ is the covariance matrix of the common factors, and $\mathbb{V}[\varepsilon_t] = \Omega$ is the covariance matrix of the idiosyncratic shocks. The first term on the right represents the systematic contribution to the total risk, while the second term is the idiosyncratic risk contribution. The same decomposition of the total assets risk applies also to a generic portfolio formed with the K assets. If we take a vector of portfolio weight ω ,³ the portfolio returns satisfy the following equalities

$$\begin{aligned} r_{p,t} &= \omega'R_t \\ &= \omega'\mathbb{E}[R_t] + \omega'\beta F_t + \omega'\varepsilon_t \\ &= \mathbb{E}[r_{p,t}] + \beta_p F_t + \varsigma_t, \end{aligned} \quad (7)$$

where $\mathbb{E}[r_{p,t}] = r_f + \beta_p\Lambda$. Moreover, we know that the total risk of the portfolio is

²This holds for any multifactor model.

³We assume that portfolio weights sum at 1 but we do not exclude short selling.

given as

$$\begin{aligned}\mathbb{V}[r_{p,t}] &= \omega' \beta \Sigma_F \beta' \omega + \omega' \Omega_\varepsilon \omega \\ &= \beta_p \Sigma_F \beta_p' + \sigma_\zeta^2\end{aligned}\tag{8}$$

This framework has relevant implications both for portfolio risk and diversification as well as for pricing of securities. If we take a diversification point of view, the final purpose is to control or sterilize the impact of asset idiosyncratic risks on the total portfolio risk. This corresponds to the willingness of achieving the following limiting condition

$$\lim_{K \rightarrow \infty} \omega' \Omega_\varepsilon \omega = \tilde{\sigma}^2 > 0\tag{9}$$

where $\tilde{\sigma}^2$ is a small quantity depending on the idiosyncratic shock variances and correlations, as well as on the portfolio composition. In a simplified setting, assuming that idiosyncratic shocks are uncorrelated, that their variances are set to an average value $\bar{\sigma}^2$ and taking an equally weighted portfolio, we have the following well-know result

$$\lim_{K \rightarrow \infty} \omega' \Omega_\varepsilon \omega = \frac{1}{K} \bar{\sigma}^2 = 0,\tag{10}$$

showing that diversification allows sterilizing the idiosyncratic shocks.

In this framework the focus is on the shocks impact, since we know that the systematic risk component cannot be diversified out, as it is driven by common factors. Therefore, in the multifactor model, the introduction of new assets allows a contraction of the contribution of the idiosyncratic component to the total risk of the portfolio, but has, in average, no effects on the systematic components.⁴

Our proposal aims at introducing in a multifactor model the impact coming from the contemporaneous links that exist across assets, when those are captured by a network. As discussed in the previous section, networks will provide information on the existence

⁴Nevertheless, we note that, by means of short selling and when a risk free asset is present, we might be able to build portfolios that annihilate the effect of at least some risk factors.

of links and might also convey details on the intensity of the link existing across assets. Therefore, we aim at coupling the systematic and idiosyncratic risks with a sort of network risk that would introduce in the model the assets cross-dependence beyond that captured by common factors. Given this further element we will then evaluate the effects on traditional uses of the multifactor model.

Let us assume that the risky assets are interconnected and that those links can be represented by a network. The network relations, as observed in the previous section, can be, in some sense, forward looking or represent the actual state of the connections across assets. From this point onward, we will assume that, indifferently from the approach adopted for the estimation of the network, the network will impact on the contemporaneous relations across assets. Starting from this assumption, we have to partially reconsider the interpretation of a general multifactor model. In fact, if we postulate the existence of contemporaneous relations across risky assets, we must acknowledge that those are not explicitly accounted for in 3. Moreover, the common factors capture the dependence of each risky asset from common sources of risk, but the presence of interconnections implies that risky assets are exposed to the movements (both systematic and idiosyncratic) of other risky assets. We might label this additional component as network exposure. In addition, risky assets might differ in terms of interconnections with other assets, and can thus be affected by an additional form of heterogeneity going beyond those associated with the different exposure to common factors and with the relevance of the own idiosyncratic risk. As a consequence, the beta matrix with respect to common factors that can be recovered from 3 cannot be directly linked to both the interconnections and to the source of *network* heterogeneity across risky assets.

One possible way of indirectly recovering the network exposure is to interpret the model in 3 as a reduced form model where reduced form parameters (the betas and the error covariance) are functions of structural parameters. The latter thus include the *true* exposure to common factors, the exposure to other assets due to the interconnections (or network exposure) and the *structural* idiosyncratic shock's variance.

To shed some light on the previous points we rewrite the model in 4 as a structural simultaneous equation system

$$A(R_t - \mathbb{E}[R_t]) = \bar{\beta}F_t + \eta_t \quad (11)$$

where the matrix A captures the contemporaneous relations across assets and it co-exist with the common factors which are here considered as exogenous variables. In 11 the covariance of η_t represents the structural idiosyncratic risk while the parameter matrix A is associated with assets interconnections, and thus with a network. Further details on the last aspect will be given in few paragraphs. If we translate the model 11 into a reduced form, we have

$$R_t = \mathbb{E}[R_t] + A^{-1}\bar{\beta}R_{m,t} + A^{-1}\eta_t \quad (12)$$

where we stress two well-known elements. Firstly, we observe that the reduced form parameters of the four factor model, which can be consistently estimated by least squares methods, are non-linear functions of the interconnections across assets (the matrix A) and of the structural exposure to common factors (the matrix $\bar{\beta}$). Secondly, the covariance matrix in 3 is also influenced by the presence of asset's interconnections. Note that, if we postulate that i) a network structure exists, and thus assets are interconnected, ii) that there are just four common factors, and then iii) we estimate the linear factor model in 3 without taking into account the network, we have by construction that the shocks are correlated.⁵ Therefore, the empirical evidences of idiosyncratic shock correlation found on the residuals of a four-factor CAPM model might be due to the exclusion of contemporaneous relations as shown by the results of Ang, Hodrick, Xing, and Zhang (2006): idiosyncratic volatility risk is priced in the cross-section of expected stock returns, a regularity that is not subsumed by size, book-to-market, momentum, or liquidity effects.

We also highlight a further aspect. If the common factors are estimated by means

⁵This holds if we assume that A is not diagonal. However, this is an inconsequential restriction as if A is diagonal we do not have contemporaneous relations across assets.

of statistical approaches rather than being observed variables, the network exposure, if present, will be totally destroyed. In fact, *statistical* factors are generally estimated from a reduced form model. Therefore, if we neglect the network exposure and adopt, say, principal component analysis, or fit a latent factor model, it might happen that one of the identified factors represent a sort of proxy of or a biased estimate of the network exposure, with possible further biases on the estimated factor loadings.

Our approach aims at re-introducing contemporaneous relations into the four-factor model thus allowing to recover both the impact of network exposure as well as the exposure to common factors. Note that both elements co-exist, and network exposure can be seen as an additional *common* risk source going beyond that of common factors. We might even define the exposure to common factors as the exogenous systematic risk exposure, while the network exposure can be labelled as an endogenous systematic risk exposure. Notably, in this way, the idiosyncratic risks will be defined as *structural* and, at least in principle, should be less correlated than the shocks in 3.

The simultaneous equation system in 11 poses serious challenges for the estimation of the matrix A . In fact, the number of factors can be assumed to be much smaller than the number of risky assets. As a consequence, to identify the structural parameters, some restrictions must be imposed on the matrix A .⁶

Our proposal for integrating network exposure and the dependence on common factors is based on the peculiar structure we give to the matrix A . We suggest to make use of an estimated network, and to specify A according to the links existing across assets as identified by the network. In our approach, the network, that represents the contemporaneous relations across assets, is used as a tool to impose restrictions on the structural parameter matrix A . In this respect, we are thus assuming that the network is given. In other words, the network is exogenous and it is used to restrict the endogenous relations in the simultaneous equation system in 11.

As we have previously argued, the network, and the associated measures of closeness

⁶Further details on identification and estimation issues are discussed in the following section.

across assets, allow us making a parallel with concepts commonly used in spatial statistics. In fact, the spatial proximity among subjects can be summarized into a weight matrix, or spatial proximity matrix, W .

This is not novel in economic applications where spatial econometrics methods have been applied in several areas including regional studies, real estate, environmental economics, international convergence and spillover; see, among many others, Elhorst (2003), Abreu et al. (2005), and Holloway et al. (2007). Nevertheless, the financial applications of spatial methods is relatively recent: Fernandez (2011) introduces a model closely related to the one we adopt, but has a focus on Value-at-Risk estimation, and determines closeness across assets on the basis of the Spearman correlation; Fernandez-Aviles et al. (2012) show that stock markets proximity should be measured by means of financial quantities and not by geographical distance; Arnold et al. (2013), combines different financial closeness measures within an equity risk management framework; Asgharian et al. (2013) analyse the impact of countries economic relation on stock market co-movements; Wied (2013) considers risk management applications and parameter stability testing on spatial models fitted to equity data; Denbee et al. (2013) focus on interbank liquidity; Keiler and Eder (2013) analyse CDS spreads attaching a systemic risk interpretation to a spatial component and defining closeness on the basis of the correlation; finally, Blasques et al. (2014) analyse spillover dynamics within a model with time-varying spatial dependence.

We differ from the previous works under different aspects. Firstly, and opposite to Keiler and Eder (2013), we interpret the spatial dependence as a component impacting on and amplifying the systematic risk component, as also affecting the idiosyncratic risks. We then go further and show how the spatial effect has an impact on both the expected returns with implications on pricing (thus getting close to the work of Fernandez, 2011), as well as on diversification. Secondly, we do not consider correlations to recover spatial dependencies but we obtain the spatial linkages from an estimated network. We thus contribute to the increasing literature providing alternative approaches for the estimation of the spatial links on the basis of an economic or financial distance. In this respect, and

in addition to the above mentioned financial works we cite, among others, the economic distance approaches adopted by Conley and Topa (2002) and LeSage and Pace (2008). Thirdly, by taking a simultaneous system estimation and an estimation of the spatial links on different databases (which also have different time frequencies) we induce a time-variation in the spatial dependence, differing but being close to the works of Keiler and Eder (2013), and Blasques et al. (2014). Fourthly, we generalize the approaches previously adopted in a financial framework allowing for asset-specific reaction to spatial links.

As we previously mentioned, in our framework spatial linkages derives from an estimated financial network. We thus assume that linked assets are neighbors. Therefore, if we have a network we can re-cast it into a sort proximity matrix W . The latter can be used to impose a structure on the matrix A . Given the matrix W , as extracted from a network, we can easily specify a spatial autoregressive (SAR) model (see Anselin, 1988, and LeSage and Pace, 2009):⁷

$$R_t - \mathbb{E}[R_t] = \rho W (R_t - \mathbb{E}[R_t]) + \bar{\beta} F_t + \eta_t \quad (13)$$

where the (scalar) coefficient ρ captures the response of each asset to the returns of other assets, as weighted with the corresponding row of W . Moreover, we assume that the error term η_t has a diagonal covariance matrix, that is $\mathbb{V}[\eta_t] = \Omega_\eta$ is diagonal. Such an assumption is required for identification purposes as we will discuss in the model estimation section. Given we assume the matrix W is known, the expected returns are conditional to the W . To maintain a simplified notation we do not report the conditioning with respect to W in the returns expectations.

At the single asset level the model reads as follows

$$R_{i,t} = \mathbb{E}[R_{i,t}] + \rho \sum_{j=1}^k w_{i,j} (R_{j,t} - \mathbb{E}[R_{j,t}]) + \bar{\beta}_i F_t + \eta_{i,t} \quad (14)$$

where $w_{i,i} = 0$, $w_{i,j} \geq 0$ and $\sum_{j=1}^k w_{i,j} = 1$. Taking a financial point of view, the

⁷Anselin (1988) calls the model mixed-regressive spatial-autoregressive. We stick here to the simpler acronym adopted in LeSage and Pace (2009).

coefficients in the vector $\bar{\beta}_i$ represent the exposure to the common factors, or *exogenous* exposure, while the coefficient ρ tracks the *endogenous* risk exposure which is influenced by the network structure, and thus called network exposure. Further insights on the interpretation of the model coefficients will be given in the following subsections.

The model in 13 can be rewritten in a more compact form as follows

$$(I - \rho W)(R_t - \mathbb{E}[R_t]) = \bar{\beta}F_t + \eta_t \quad (15)$$

thus highlighting the fact that spatial proximity and the associated SAR model give a structure to the contemporaneous relation matrix, which is now parametrized as

$$A = I - \rho W \quad (16)$$

The structural model now includes contemporaneous relations, driven by links or connections across asset, systematic components and asset specific shocks.

The model in (15) has, however, a very restricted structure. In fact, there is a single parameter, the ρ driving the network exposure. This can be easily generalized by allowing for asset-specific responses to the network structure. We can thus modify the contemporaneous relation matrix of (16) into

$$A = I - \mathcal{R}W \quad (17)$$

where $\mathcal{R} = \text{diag}(\rho_1, \rho_2, \dots, \rho_K)$ is a diagonal matrix. This model is similar to the fixed coefficient specifications for spatial panels discussed in Elhorst (2003). A clear advantage of such a structure is given by the possibility that assets have different network exposures, as for each asset the model becomes

$$R_{i,t} = \mathbb{E}[R_{i,t}] + \rho_i \sum_{j=1}^k w_{i,j} (R_{j,t} - \mathbb{E}[R_{j,t}]) + \bar{\beta}_i F_t + \eta_{i,t}. \quad (18)$$

To estimate the asset-specific parameters the network must satisfy an identification condition: each asset must be connected to at least one other asset. If this is not the case, the diagonal of matrix \mathcal{R} must be restricted in such a way that not-connected assets will not have a network exposure. Further details will be discussed in the estimation section.

The spatial econometrics literature generally assumes that the spatial proximity matrix is time invariant. In fact, if the matrix W depends on physical measures, such as those is the space, those can be safely assumed constant over time. However, in a financial framework, the connections across assets might change over time for a number of reasons, some of them being, for instance, the occurrence of an unexpected market shock, mergers and acquisitions. Similar approaches have been adopted by Asgharian et al. (2013) and Keiler and Eder (2013). We mentioned in Section 2 that the network structure can be estimated on the basis of different approaches and data. The latter can be either time series and/or cross sectional data. Therefore, the networks might be estimated, with the same type of data, over different samples. An example of this approach will be provided in the empirical section. Clearly, by changing the sample, we can easily obtain different networks, and the time-evolution of connections across assets is itself a relevant, but also expected, finding. Despite the time-variation of the networks, and still assuming the network exogenous with respect to the linear structural model,⁸ the contemporaneous matrix can be further re-written as

$$A_t = I - \mathcal{R}W_t \quad (19)$$

where we highlight that the network changes over time, and thus lead to a time-varying W matrix. In turn, this induce time-dependence on the A matrix, as well as on the reduced

⁸We might relax the exogeneity assumption by stating that the network are known conditionally to the past.

form parameter matrices, both on the betas as well as on the covariance of idiosyncratic shocks, that is, we have also heteroskedasticity. Nevertheless, we might postulate that the dynamic of W_t is smooth, and operates at lower time scales as compared to those monitoring the evolution of returns (for instance we can assume the W matrices change over years, or after specific events such as crises). Therefore, the heteroskedasticity is mild, and the betas are slowly evolving. The use of time-varying W matrices thus lead to a time change in the spatial dependence differing from the approach of Blasques et al. (2013) that obtain the same result by letting the \mathcal{R} parameters being time-varying. We notice that, if the network exposure exist and the structural parameters in the matrix $\bar{\beta}$ are constant, the estimation of the reduced form model over different sample might suggest changes in the factor exposure. However, those changes are not present but due to the misspecification of the network relations. We remind that the expected returns are conditional to the W matrix. If the network exposure is time-varying, the conditioning will operate over the full history of the time-varying W_t .

We close this section by introducing a further generalization of the model which is both intuitive and feasible. This refers to the possibility of constructing a network structure from different data, for instance cross-exposures or estimation of causality relations..

A-priori, we do not have information allowing to order the alternative networks. However, those can be easily introduced in the model, allowing the data to give an answer. In fact, the contemporaneous relation matrix can be written as

$$A = I - \sum_{j=1}^m \rho_j W_j \quad (20)$$

where m different networks are jointly introduced into a model. The estimated parameters can then provide useful details on the relevance/preference of different network measures.⁹ We also note that distance matrices W recovered from a network approach

⁹We note that, when the network exposure parameter are asset-specific, the introduction of different W matrices requires some identification conditions that depend on the network structures.

can be also jointly used with similar matrices obtained from different methods, such as on the basis of economic sector partitions of assets as in Arnold et al. (2013) and Caporin and Paruolo (2013), bilateral trades (Asgharian et al. 2013), or foreign direct investments (Fernandez-Avila et al. 2012).

3.1 The impact of common factor and networks

The reaction of one asset to common factors and network exposure appears in a more clear way once we rewrite the model in a reduced form representation, highlighting the impact of the network connections included in W on the reduced form parameters (the reduced form betas and the reduced form shock's covariance):

$$R_t = \mathbb{E}[R_t] + \mathcal{A}\bar{\beta}F_t + \mathcal{A}\eta_t \quad (21)$$

where $\mathcal{A} = A^{-1}$, $A = I - \rho W$ and we assume that A is non-singular. For simplicity, we focus on the case where the network exposure is driven by a single parameter, the ρ . However, all derivations and comments apply also to the more general parametrizations of the matrix A previously introduced.

From LeSage and Pace (2009) we take the following relation

$$(I - \rho W)^{-1} = I + \rho W + \rho^2 W^2 + \rho^3 W^3 \dots, \quad (22)$$

where the term ρW monitors the effect of linked assets (in spatial econometrics, the neighbours), for instance if asset j is linked to asset i we have a non-null entry in W_{ij} . Differently, $\rho^2 W^2$ is associated with the effect on asset j induced by the assets linked to asset i (those called in spatial econometrics, the second-order neighbours). The latter relation can be further generalized to higher orders. Notably, the matrices W^j might also include a so-called *feedback loop* as, following the previous example, asset i can be linked to asset j (the relation is thus bi-directional), causing the matrix W^j to have non-null elements on the main diagonal. We stress that, despite the summation has infinite terms,

by imposing that $|\rho| < 1$ we ensure the effect of linked assets converges to zero. On the contrary, if $|\rho| > 1$ we might have explosive patterns.

By using 22 we can rearrange the model in 21 as

$$R_t = \mathbb{E}[R_t] + \bar{\beta}F_t + \sum_{j=1}^{\infty} \rho^j W^j \bar{\beta}F_t + \eta_t + \sum_{j=1}^{\infty} \rho^j W^j \eta_t. \quad (23)$$

Such a representation highlights that the impact of the common factors as well as of the idiosyncratic shocks on the risky asset returns can be decomposed into two parts (each including two terms). The first component is the traditional, or direct, or structural impact, while the second component is the impact associated with the network exposure. We can thus define the following four elements:

- a - $\bar{\beta}F_t$: the structural exposure to common factors;
- b - $\sum_{j=1}^{\infty} \rho^j W^j \bar{\beta}F_t$: the network exposure to common factors;
- c - η_t : the structural impact of idiosyncratic shocks;
- d - $\sum_{j=1}^{\infty} \rho^j W^j \eta_t$: the network impact of idiosyncratic shocks.

Note that the network-related exposures depends on the structure of the matrix W as well as on the parameter monitoring the network impact, the ρ . A relevant remark comes from the network impact of common factors. Let's take for simplicity a specific common factor, that is, we focus on a single column of F_t and consider the impact of the m -th factor on the risky asset returns

$$\bar{\beta}_m + \sum_{j=1}^{\infty} \rho^j W^j \bar{\beta}_m. \quad (24)$$

Equation (24) provides two relevant insights.

At first, we note that the network exposure to common factors acts as a multiplier of the structural exposure if the ρ coefficient is positive (W elements are anyway positive). Therefore, shocks to the common factors will be amplified by: the presence of connections

across assets, that is when, for asset i , the i -th row of W contains at least one non-null element; the change in the impact of network connections, that is when the ρ coefficient increases; by changes in the network structure, that is when the matrix W changes. Note that, if asset i is not connected to other assets, all products $\rho^j W^j \bar{\beta}_i$ are equal to zero.

Now assume that for the risky asset i the m -th common factor is not relevant (that is $\bar{\beta}_{i,m} = 0$). In this case, in the standard linear factor models, the common factor will have no role in explaining the asset returns. However, when asset are linked and network exposures are taken into account, a common factor to which a risky asset has a zero structural exposure might still be relevant to explain the risky asset return evolution. Such an effect is not direct but induced from the network exposure and is associated with the existence of non null elements in the i -th row of the matrix W . Take for instance the following case

$$W = \begin{bmatrix} & \vdots & & \\ \mathbf{0}_i & 1 & \mathbf{0}_{K-i-1} & \\ & \vdots & & \end{bmatrix} \quad (25)$$

where assets i is connected only to asset $i + 1$ and subscripts denote the length of row vectors of zeros. Moreover, assume the following factor exposure for both assets

$$\bar{\beta} = \begin{bmatrix} & \vdots & & & \\ & \beta_{M,i} & 0 & 0 & 0 \\ & \beta_{M,i+1} & \beta_{SMB,i+1} & 0 & 0 \\ & \vdots & & & \end{bmatrix}, \quad (26)$$

where asset i is not exposed to SMB while asset $i + 1$ is affected by the same risk factor. Asset i dependence on risk factors can thus be represented as

$$\beta_{M,i} R_t^M + \rho \beta_{M,i+1} R_t^M + \rho \beta_{SMB,i+1} R_t^{SMB} + \sum_{j=2}^{\infty} (\rho^j W^j \bar{\beta} F_t) |_i \quad (27)$$

where $|_i$ identifies the i -th element of a vector. Note that the last term on the right

represents further elements that can be specified only through the knowledge of the entire W matrix. Therefore, even if a risky asset i is not (structurally) exposed to a common factor (in the previous example SMB), the common factor will play anyway role if it impacts on the returns of the assets to which i is linked.

Such a result can be further generalized by focusing, for instance, on sector specific risk factors. Those, in presence of a network exposure, despite being sector specific will have a systematic impact on all connected assets. Moreover, if we disregard the network exposure, we might also incur in the risk of misinterpreting the impact of risk factors. In fact, by estimating the reduced form model we might label as *common* a factor that in reality is structurally related just to a subset of the investment universe and impact on other assets only through network connections.

A similar property exists for the idiosyncratic shocks. In fact, if we assume they are uncorrelated, the existence of network connections implies that the structural shocks of one asset impacts on the returns of all the connected assets. Therefore, shocks on single assets can have effects on many other risky assets.¹⁰

From a pricing perspective, the expected returns in the most general model specification equals

$$\mathbb{E} \left[R_t | \{W_t\}_{t=1}^T \right] = r_f + \bar{\beta}\Lambda + \sum_{j=1}^{\infty} \mathcal{R}^j W_t^j \bar{\beta}\Lambda, \quad (28)$$

thus highlighting the relevance of network exposures, that impacts on the expected returns (which are conditional to the sequence of the W_t matrices). The existence of links across assets induces higher expected returns as opposed to the case where links are absent; this comes from the fact that we postulate the coefficients in \mathcal{R} and the elements of W_t are all positive. Thus, the exposure to common factors might depends on the

¹⁰Summary measures of the exposure to common factors and idiosyncratic shocks can be obtained by mimicking the approaches used in spatial econometrics. A discussion on this topic is included in LeSage and Pace (2009), see their section 2.7; these measures have been used in a financial framework by Asgharian et al. (2013). We also note that the decomposition of asset returns into four elements is equivalent to that of Abreu et al. (2005) for separating the standard impact of covariates from that due to the spatial links, and is thus an alternative to the impact measures of LeSage and Pace (2009).

connections with other assets and, bearing in mind the previous discussion, the expected returns might depend on risk premiums associated with factors to which a given asset is not directly exposed. The heterogeneity with respect to connections creates reactions to shocks on the common factors that are different across assets due to the different exposures of assets to the factors, but also due to the different impact of feedback loops coming from the underlying network structure. The change over time of the W_t matrix, or the presence of a structural break on the \mathcal{R} coefficients (that we might locate in proximity of a crisis or of an extreme event) might create abrupt changes in the expected returns with the consequence of relevant movements in stock prices.

In addition, we stress that the use of a network which is very dense, thus implying a W matrix almost full, will have further impacts. In fact, a full W implies that all idiosyncratic shocks are correlated. However, from our viewpoint, this corresponds to an indirect evidence of model misspecification as an additional common factor is now present but not taken into account. As a consequence, such a common factor risk must be priced, and could generate the empirical evidences shown by (?). The latter case could also correspond to an empirical evidence challenging the validity of the APT pricing approach. From a different viewpoint, our modeling framework still satisfies the assumptions required for APT. As we will show in the next section, the presence of a network exposure despite inducing correlation across the idiosyncratic shocks does not exclude the existence of diversification benefits. In turn, this is sufficient to guarantee the validity of the APT where risk premiums can be recovered from the reduced form model. Finally, when we introduce a time-variation in the W matrices, or in the \mathcal{R} elements, the APT still holds but with risk premiums estimated in the cross-sectional dimension for fixed values of W and/or \mathcal{R} and thus inducing also a time-change in the factor risk-premiums.

Up to this point, we have not yet discussed the sign of the ρ coefficient. Intuitively, we expect that the assets are positively related one to the other, as the links are coming from a network. We thus imagine that shocks transmit to connected assets preserving their sign. If we take a simplified model with one single ρ coefficient, it is highly improbable

we will ever observe negative coefficients. In fact, a single coefficient represents a sort of average reaction of the asset to the shocks coming from neighbors. However, in a model accounting for the heterogeneity of the reaction to the network exposure, negative asset-specific coefficients might appear. In other words, we cannot exclude a-priori that a shock in one asset lead to an apposite movement of a linked asset. We motivate such a finding by making a parallel with negative correlations. If two assets are negatively correlated, their joint introduction in a portfolio lead to a decrease of the overall variance as compared to the case in which only one of the two assets were present. In a factor model, negative correlations across asset returns can be motivated by loadings to the (same) common factors having different signs. In our framework, negative correlations across asset returns can emerge both in response to different signs in the factor loadings but also due to the presence of negative asset-specific reaction to the network exposure. Consider the reduced form of our model as represented in equation (21). In this case, the innovation term has a non-diagonal covariance. Let's also assume that the spatial proximity matrix W is time invariant and thus the reduced form model has time invariant betas and homoskedastic innovations. If we estimate the reduced form model, the innovations could show evidence of non-null correlations, some of them being negative. They can be due both to the presence of opposite reaction to the common factors but also due to the presence of negative ρ_i coefficients. Within our model, negative ρ might thus exist, but how can we interpret them from a pricing perspective? We read them as evidences of risk absorption due to the network exposure. In fact, a negative ρ_i allows a reduction of the exposure of one asset to the common factors, since the $i - th$ component of the second term in equation (24) becomes negative. However, the consequence of this risk absorption is also a reduction of the contribution of network exposure to the expected return. In fact, also the $i - th$ component of the third term in equation (28) will become negative.

We further note that the use of a matrix $A = I - \mathcal{R}W$ lead to a focus on the impact of the network exposures where the asset-specific coefficients ρ_i represents the impact on i coming from the assets linked to i , or, from a different viewpoint, it is the loading of i

from the network risk. We might, however, be interested on the effect of asset i on the other assets, having thus a ρ_i coefficient that represents the impact of i to the assets to which i is linked. We might see this as an outgoing effect of i to other assets through the network, or as a contribution of i to the network factor.

This can easily be achieved with a simple modification of the model, by replacing A with $B = I - W\mathcal{R}$. With such a change, the return equation (18) becomes

$$R_{i,t} = \mathbb{E}[R_{i,t}] + \sum_{j=1}^k w_{i,j} \rho_j (R_{j,t} - \mathbb{E}[R_{j,t}]) + \bar{\beta}_i F_t + \eta_{i,t}. \quad (29)$$

We now note that the ρ_j coefficients represent the impact of the j -th asset on the other assets. Moreover, if we consider the reduced form representation of the model, we have

$$R_t = \mathbb{E}[R_t] + \mathcal{B}\bar{\beta}F_t + \mathcal{B}\eta_t \quad (30)$$

where $\mathcal{B} = B^{-1}$. The reduced form betas can again be seen as a by-product of both the structural risk exposure, the $\bar{\beta}$ and the inflating factor coming from the network, the \mathcal{B} . However, the structure of \mathcal{B} has a different interpretation. In fact, the coefficients are no more linked to the loading of the network risk but rather to the effect a given asset is causing to other assets or to its contribution to the network risk.

3.2 Risk decomposition

The model in 11 allows recovering a risk decomposition similar to that available for the standard linear factor models in 2. The starting point is the reduced form introduced at the beginning of the previous subsection, see 21. Equation 21 highlights that the estimation output of standard multifactor models can be coherent with the presence of contemporaneous links across assets. In fact, we can redefine $\beta = \mathcal{A}\bar{\beta}$ and $\varepsilon_t = \mathcal{A}\eta_t$, and estimate the reduced form mean parameters, the matrix β and the covariance of ε_t . However, this does not lead to the identification of the structural parameters: the structural

factor loading $\bar{\beta}$, and the network related coefficient ρ included in \mathcal{A} . On the contrary, our purpose is to identify structural parameters of 11. Given the knowledge of structural parameters, the total variance of the risky assets can be written as follows

$$\mathbb{V}[R_t] = \mathcal{A}\bar{\beta}\bar{\beta}'\mathcal{A}'\sigma_m^2 + \mathcal{A}\Omega_\eta\mathcal{A}' \quad (31)$$

Despite being equivalent to the traditional risk decomposition of a multifactor model, 31 provides a relevant insight. In fact, both the systematic and idiosyncratic risk components are influenced by the presence of interconnections across assets as the matrix \mathcal{A} appears on both the right hand side terms. This shows also that, if we estimate the reduced form model with standard linear methods, our evaluations of the systematic and idiosyncratic risk components are in reality a blend of the structural loadings and idiosyncratic risks with the network relations. Keiler and Eder (2013) suggest that the presence of spatial links could be interpreted as a systemic risk contribution. However, the previous decomposition provides an alternative view, where spatial dependence is not an additive source of risk but rather a multiplicative one, where the asset-specific effect cannot be easily recovered (as it depends on both the structure of the network and the associated W matrix and the spatial parameters in \mathcal{R}).

Obviously, the same structure appears at the portfolio level where we have

$$\mathbb{V}[r_{p,t}] = \omega'\mathcal{A}\bar{\beta}\Sigma_F\bar{\beta}'\mathcal{A}'\omega + \omega'\mathcal{A}\Omega_\eta\mathcal{A}'\omega \quad (32)$$

Since our main focus is a portfolio of risky assets, we start elaborating on the last decomposition of the portfolio total risk. Nevertheless, we stress that comments similar to those later reported apply also to each risky asset return. We assume that we want to maintain a reference with the structural parameters $\bar{\beta}$ as they represent the impact of systematic movements on the portfolio. However, the existence of interconnections across assets is affecting such impact at the portfolio level, moving it away from that we would

have observed if interconnections were not be present. The latter, common factor loading without interconnections, would equal $\omega'\beta$, but in reality, i.e. with interconnections, we have $\omega'\mathcal{A}\bar{\beta}$. We might thus interpret the product $\omega'\mathcal{A}$ as a transformation of portfolio weights, due to the impact of the interconnections across assets. The factor \mathcal{A} amplifies or reduces the relevance of one asset compared to its true monetary weight in the portfolio. Those modified weights represent the impact at the portfolio level of systematic shocks affecting the risky assets. The interconnections are thus matched with the portfolio weights rather than altering the betas. This is just a choice which we further motivate by the decomposition we now introduce.

We first note that, if assets interconnections are not present (that is when $\mathcal{A} = I$), the idiosyncratic risk equals Ω_η while the systematic risk component is $\bar{\beta}\Sigma_F\bar{\beta}'$. We rewrite portfolio variance decomposition in 32 by adding and subtracting the portfolio idiosyncratic and systematic variance components when those are not influenced by asset interconnections:

$$\mathbb{V}[r_{p,t}] = \omega'\mathcal{A}\bar{\beta}\Sigma_F\bar{\beta}'\mathcal{A}'\omega + \omega'\mathcal{A}\Omega_\eta\mathcal{A}'\omega \pm \omega'\bar{\beta}\Sigma_F\bar{\beta}'\omega \pm \omega'\Omega_\eta\omega \quad (33)$$

After rearranging, the total portfolio variance can be recast into a decomposition counting four different terms

$$\mathbb{V}[r_{p,t}] = \underbrace{\omega'\bar{\beta}\Sigma_F\bar{\beta}'\omega}_I + \underbrace{(\omega'\mathcal{A}\bar{\beta}\Sigma_F\bar{\beta}'\mathcal{A}'\omega - \omega'\bar{\beta}\Sigma_F\bar{\beta}'\omega)}_{II} \quad (34)$$

$$+ \underbrace{\omega'\Omega_\eta\omega}_{III} + \underbrace{(\omega'\mathcal{A}\Omega_\eta\mathcal{A}'\omega - \omega'\Omega_\eta\omega)}_{IV} \quad (35)$$

We give the following interpretation to the four risk components:

I Is the structural systematic risk component that depends on the structural loadings from the common factors and from the covariance of the common factors; this is the

exogenous systematic effect;

II Is the effect of asset interconnections on the systematic risk component, or first contribution of network exposure to the total risk; this is the *endogenous* systematic effect;

III Is the structural idiosyncratic component that depends only on the structural shocks covariance;

IV Is the effect of interconnections on the idiosyncratic risk, or second contribution of network exposure to the total risk; this might be interpreted as an *endogenous* amplification of idiosyncratic risks.

Note that by adding the second and fourth terms we obtain the total contribution of network exposure to the total portfolio risk. We finally notice that the model with assets interconnections gives the standard multifactor model if there are no interconnections, that is W is a null matrix, or, if the coefficient ρ is statistically not significant.

In addition, the network exposure impact on the idiosyncratic part of the variance implies that the diversification benefits might be endangered depending on the network structure. In fact, despite the fourth term will decrease with increasing cross-sectional dimension, the decrease speed will be smaller compared to the case without network effects.

Similarly to the standard linear factor model, we can recover analytical elements in a simplified setting. As we previously stated, the covariance matrix Ω_η is diagonal; we further assume that the diagonal elements are set to an average value $\bar{\sigma}^2 = 1$. In addition, we take an equally weighted portfolio, consider the existence of a single coefficient ρ for all asset, and take a limiting case where all assets are connected (thus W has zeros only over the main diagonal, while off-diagonal terms equal $\frac{1}{K-1}$ after row-normalization). In this case, it can be shown that

$$\begin{aligned}
\omega' \mathcal{A} \Omega_{\eta} \mathcal{A}' \omega &= \bar{\sigma}^2 \omega' \mathcal{A} \mathcal{A}' \omega & (36) \\
&= \frac{\bar{\sigma}^2}{K^2} \mathbf{i}'_K \mathcal{A} \mathcal{A}' \mathbf{i}_K \\
&= \frac{K + \rho^2}{(K + \rho)^2 (\rho - 1)^2} \bar{\sigma}^2
\end{aligned}$$

where K is the asset number and \mathbf{i}_K is a K -dimensional vector of ones.¹¹ Moreover, we have that

$$\lim_{K \rightarrow \infty} \frac{K + \rho^2}{(K + \rho)^2 (\rho - 1)^2} \bar{\sigma}^2 = 0 \quad (37)$$

thus preserving the diversification benefit. However, the idiosyncratic risk contribution is higher than in the case without spatial dependence (i.e. with $\rho = 0$). In fact, we can show that the above reported portfolio idiosyncratic risk is higher than $\frac{1}{K} \bar{\sigma}^2$ thus confirming that term IV is positive. Figure (1) provides a graphical example with different ρ values of the portfolio idiosyncratic risk across different portfolio sizes.

As we noted in the previous subsection, the elements of the matrix \mathcal{R} can be also negative, thus leading to negative correlations across asset returns. In that case, the negative ρ_i play a role equivalent to negative correlation thus absorbing a portion of the systematic or idiosyncratic risks. As a consequence, in a general model with heterogeneous asset reaction to the network exposure, the components II and IV in the risk decomposition we have just introduced, can also become negative. In such a case, the network exposure reduces risk, and this could also be seen as a kind of flight-to-safety effect: if shocks hit financial assets and then transmit to industrial pro-cyclical sectors, we cannot exclude that the anti-cyclical sectors will anyway suffer.

The previous model gives thus a framework where we can analyse the impact at the

¹¹In the special case considered the diagonal elements of \mathcal{A} equals $\frac{(K-1)\rho-K}{\rho^2+(K-1)\rho-K}$ and the off-diagonal elements are $\frac{-\rho}{\rho^2+(K-1)\rho-K}$. Moreover, the diagonal elements of $\mathcal{A}\mathcal{A}$ equal $\frac{K\rho^2+[(K-1)\rho-K]^2}{[\rho^2+(K-1)\rho-K]^2}$ and the off-diagonal are $\frac{(K-1)\rho^2-2\rho[(K-1)\rho-K]}{[\rho^2+(K-1)\rho-K]^2}$. Summing up the elements in $\mathcal{A}\mathcal{A}$ and simplifying we obtain the above reported result.

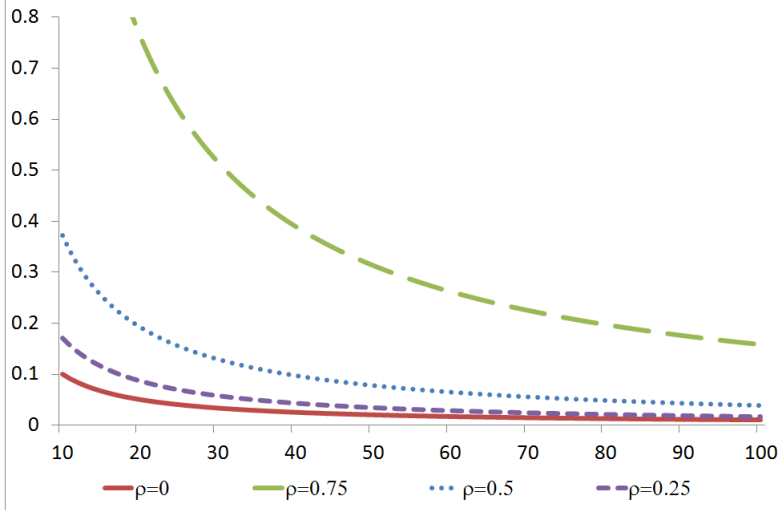


Figure 1: Portfolio idiosyncratic risk across different ρ levels and increasing number of assets. The case $\rho = 0$ corresponds to the absence of spatial links and is the standard result for diversification benefits.

portfolio level of the interconnections we might observe across assets, and how those interconnections can endanger/limit the benefits of portfolio diversification. The following section provides some further evidences, examples and comments on a simulated framework.

3.3 Simulation example

To give an idea of the ability of this proposed framework to provide insightful features and mostly of the relevance of the parameter ρ , we simulate 100 different stocks returns that evolve according to a single factor model. The common factor has a volatility of 15% per year and its return is set to 0%. Moreover, the structural betas of the 100 stocks loading the common factor movements have been extracted from a uniform distribution $U(0.8; 1.2)$, and the idiosyncratic volatilities have been randomly generated from a uniform distribution $U(20\%; 40\%)$. Then, we consider three different spatial matrices W :

- W_1 , the Market Matrix, which is the spatial matrix where each asset is linked to all the others so that the network is fully connected. Since, by definition, the

principal diagonal of the spatial matrix contains only zeros, the market matrix equals

$$W_1 = 1_k 1_k' - I_k;$$

- W_2 , the Two-Neighbours Matrix, which is a tri-diagonal matrix (with zeros on the main diagonal) where each asset has only two neighbours (assets 1 and K have only a single neighbour);
- W_3 , the Random Matrix, where each element of the matrix, main diagonal excluded, is extracted from a Bernoulli density with probability set to $p = 0.3$.

Note that matrices W_1 and W_2 are symmetric while matrix W_3 is not symmetric.

In the presence of asset connections, the variance and risk depend not only on how the spatial effect spreads among the assets or in other words, on the network structure, but also on the parameter monitoring the network (or spatial) impact, that is the value of ρ or \mathcal{R} . Therefore, in order to shed some light on the relation between those two elements, we take into account increasing values of ρ , starting from 0, thus absence of spatial links, to 0.25, 0.5 and 0.75.

We start from the analysis of returns and consider the impact of network connections on the common factor exposure. We thus compare the structural betas $\bar{\beta}$ with the betas augmented by the presence of links across assets, that is

$$\mathcal{A}\bar{\beta} = \bar{\beta} + \sum_{j=1}^{\infty} \rho^j W^j \bar{\beta}. \quad (38)$$

As we have extracted betas from a uniform, to simplify the graphical representation we order assets with respect to the values of $\bar{\beta}$ and separately report in Figures (2) and (3) the structural beta and the increases in the betas induced by different values of ρ and different spatial matrices W .

We observe that, when the spatial matrix changes, the effect is clearly different across assets depending on the network structure, but the average of the betas is almost the same for all the three W . On the contrary, changes to the coefficient ρ will lead to substantial modifications of the betas; with increasing levels of ρ the network impact

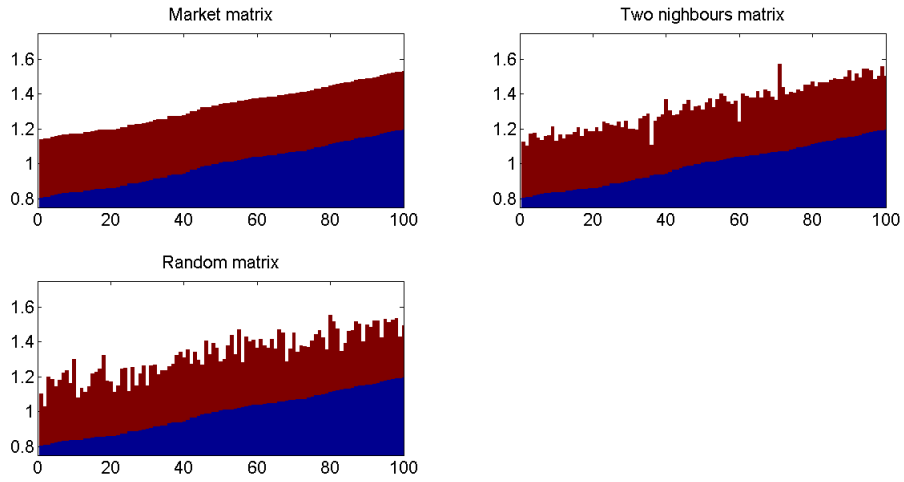


Figure 2: Beta values across assets: structural betas (in blue) and augmented betas (in red) across different spatial matrices W .

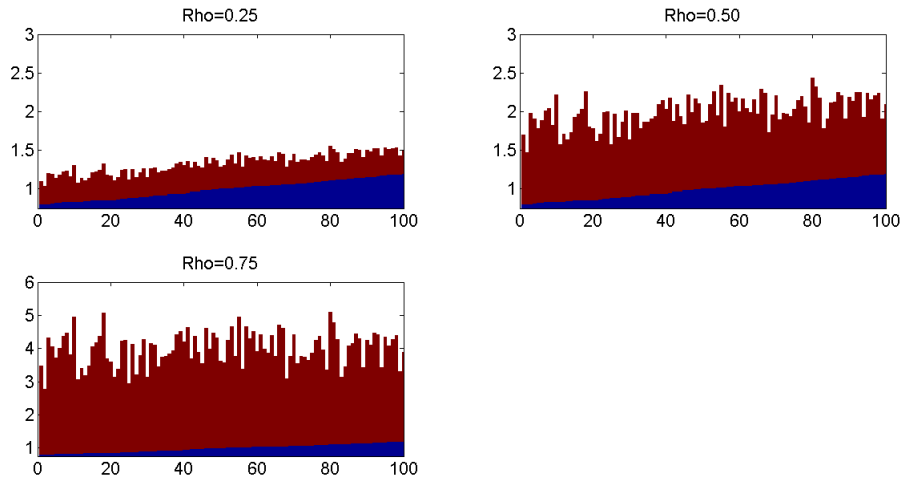


Figure 3: Beta values across assets: structural betas (in blue) and augmented betas (in red) across different values for ρ with the random matrix W .

tends to increase exponentially and beta are significantly larger higher is the ρ having significant implications on the risk premia. In fact larger beta because of higher network exposures (i.e. higher ρ) means a larger risk premia.

We then focus on the variance decomposition for each simulated asset and, in particular, we analyse the role on the total variance of the four components presented above. The main purpose is to show how relevant is the effect of the interconnections on the total variance of the assets. By the model previous introduced we are able to determine the impact network interconnections have on the exposures to systematic and the idiosyncratic components. More specifically, we show here how the network exposure impact on the variance decomposition implied by a factor model

Our model captures network exposures by means of the spatial parameter ρ (or by the matrix \mathcal{R}). We then starts from a ρ equal to zero, implying that the matrix A of simultaneous link (network interconnections) becomes an identity matrix. Consequently, the model collapses to the standard general APT/multifactor model, and the variance components are just two: the systematic and the idiosyncratic, see Figure (4).

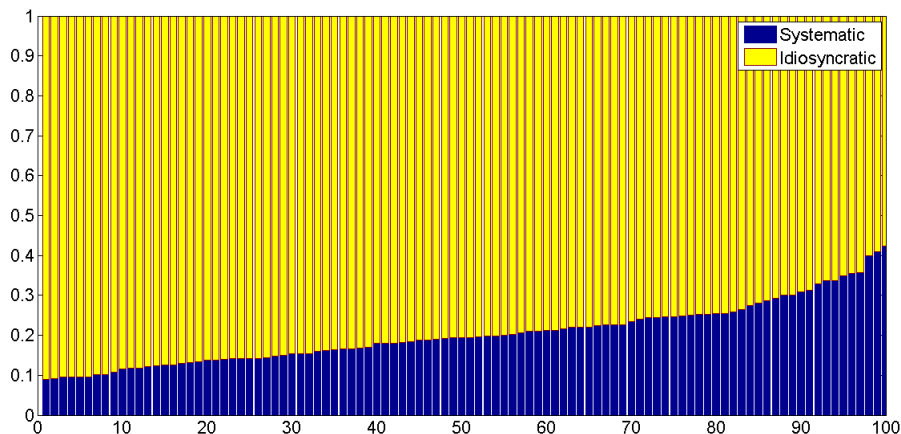


Figure 4: Relative variance decomposition with no spatial interaction $\rho = 0$, the model corresponds to general multifactor model.

If ρ takes positive values, it means that there is second relevant source of risk: assets returns are characterized by network interconnections. This effect changes the composition of the asset's variance as we have shown above.

Using the market matrix W_1 and $\rho = 0.25$, for each asset the variance has three relevant components: the systematic component, the idiosyncratic component and the network impact on the systematic component. The network effect on the idiosyncratic components is almost close to zero and thus not relevant. Similar results are obtained by increasing the value of the ρ coefficient. Therefore, if all assets are neighbours, we have that the network connections impact almost only on the systematic component.

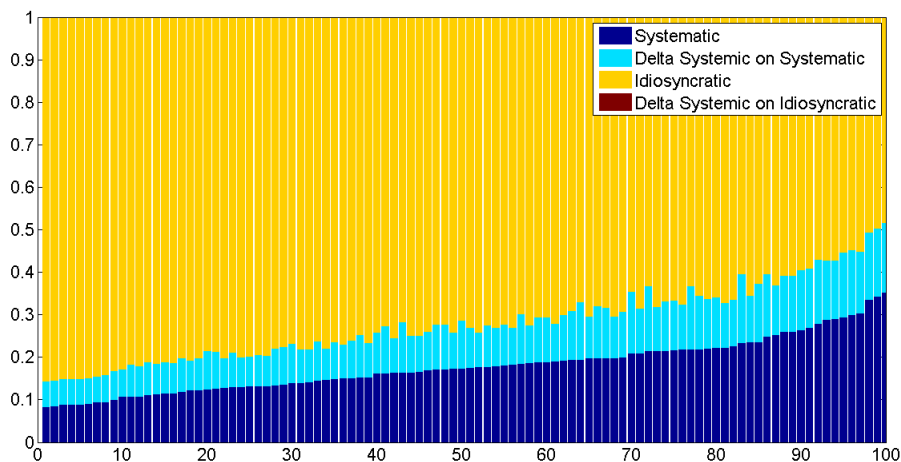


Figure 5: Relative variance decomposition with spatial interaction $\rho = 0.25$ and Spatial Matrix W_1 , we note three relevant components the systematic component, the idiosyncratic component, and the network impact on the systematic component; network impact on the idiosyncratic component is tiny and not visible in the plot.

If we substitute the Market matrix with Two-neighbours matrix while still maintaining the $\rho = 0.25$, we can observe for each asset variance the presence of the four components: the first two standard (systematic and idiosyncratic) terms as well as the two network impacts on the systematic and idiosyncratic components. With respect to the previous case, the change in the structure of the system, the network connections and the way shocks spread across the network, has a relevant effect. When this is combined with the impact of network connections on the risk, we see that the system becomes more vulnerable. In fact, even if the spatial parameter is the same of the previous case, $\rho = 0.25$, the network impact on asset variances is much higher.

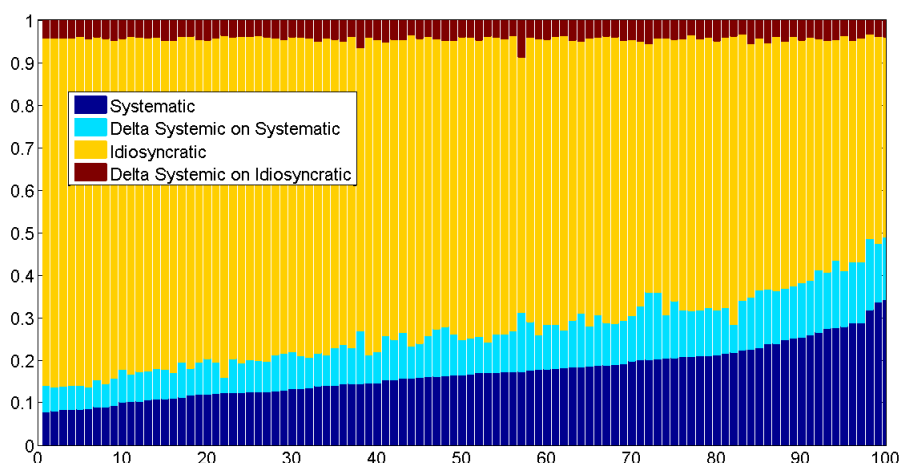


Figure 6: Relative variance decomposition with spatial interaction $\rho = 0.25$ and Spatial Matrix W_2 , we distinguish four components the Systematic component, Idiosyncratic component, and the spatial effect on the systematic component and the spatial Effect on the idiosyncratic component

Using $\rho = 0.5$ and random matrix, we distinguish four sources of risks, as in the previous case, but the spatial effect is clearly prominent.

In the simulation above we have investigated the effect of ρ , and W_i on the variance of each asset. To investigate the impact of ρ , and W_i on portfolio variance and diversification we construct an equal weighted portfolio with the 100 assets. We compute the variance for increasing spatial effect ρ , using the Random Matrix. We observe that the spatial effect on the systematic and idiosyncratic components for the portfolio variance becomes prominent as soon as ρ increases (see figure 8).

As in Ross (1976), whenever we hold a portfolio with very large number of asset, the variance of the idiosyncratic component tends to assume very small value. The principal results of our simulations is that the spatial interaction affects the way to diversify the risk on the portfolio, in particular for high values of ρ i.e 0.75, the idiosyncratic part of the variance of equal weighted portfolio assumes higher value than the case with no spatial interactions for the same number of assets held (see figure 9), in accordance with the graphical example of the previous section.

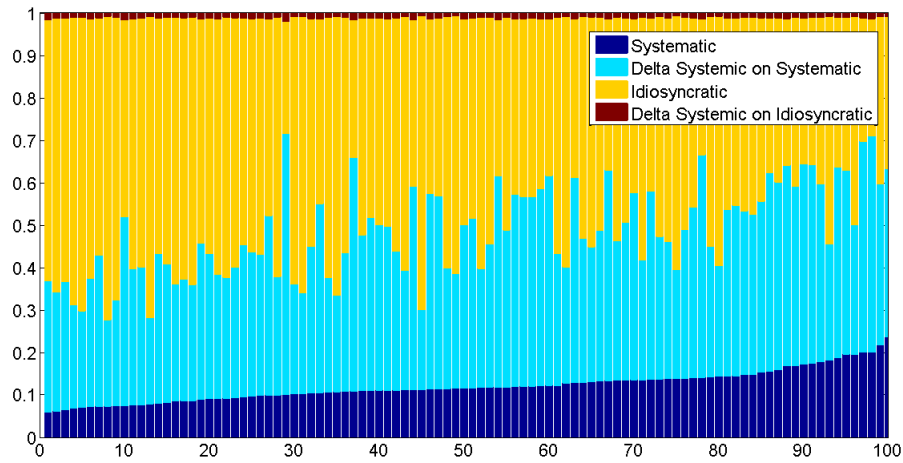


Figure 7: Relative variance decomposition for each asset with spatial interaction $\rho = 0.5$ and Spatial Matrix W_3 "Random Matrix". We distinguish four components the Systematic component, Idiosyncratic component, and the spatial effect on the systematic component and the spatial Effect on the idiosyncratic component, in this case the spatial effect becomes relevant

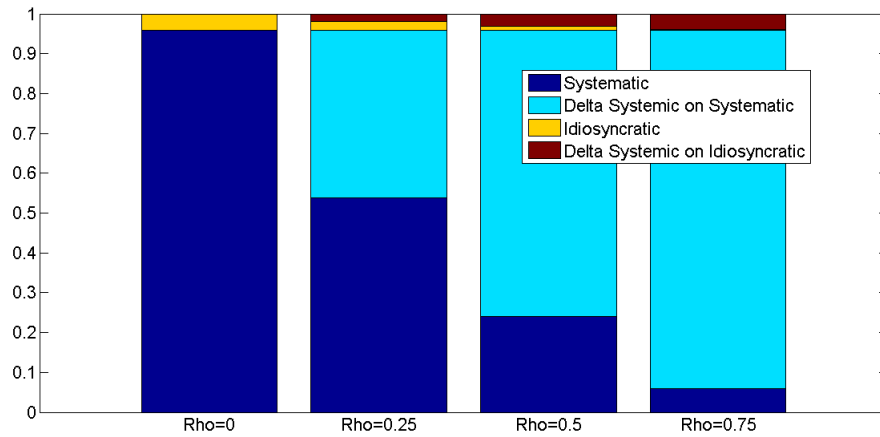


Figure 8: Relative variance of equal weighted portfolio with increasing spatial parameters ρ

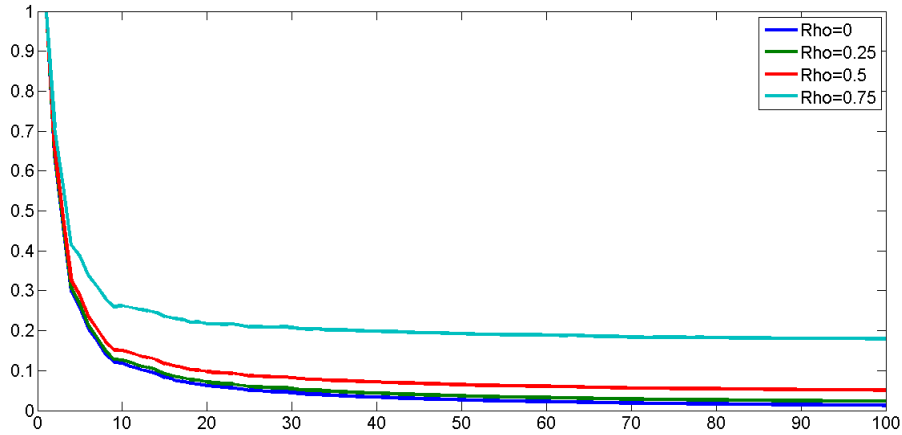


Figure 9: Relative variance of equal weighted portfolio with increasing spatial parameters ρ

In summary we have described the effect of different network interconnections and exposures on asset and portfolio volatility.

The next session we present the estimation methodology needed to apply the model presented above to market data.

4 Model estimation

We have seen how to interpret the model parameters and how to derive from the models intuitive decomposition both on the returns as well as on the total risks. However, model parameters must be estimated and this poses relevant challenges. Let us report the simultaneous model equation

$$AR_t = \alpha + \bar{\beta}F_t + \eta_t. \quad (39)$$

As standard econometrics textbook reports, identification conditions are required to estimate the parameters of A , α , $\bar{\beta}$ and $\mathbb{V}[\eta_t]$. The simple order condition of identification requires that the model parameters must be less than the parameters we can recover from the reduced form specification. In fact, the latter can be estimated by least square

methods, and structural parameters could be recovered thanks to their relation with reduced form parameters. The reduced form model is

$$R_t = \alpha^* + \bar{\beta}^* F_t + \epsilon_t. \quad (40)$$

suggesting we can consistently estimate $4K$ mean parameters plus $\frac{1}{2}K(K+1)$ covariance parameters. However, an unrestricted structural specification, despite having the same number of parameters in the covariance, has $4K + K^2$ mean parameters.

The presence of assets interconnections, summarized into a network, allows a sensible reduction of the number of parameters included in the matrix A . In fact, if we have asset-specific network exposures and a single network, we have only K parameters in A . However, this is not sufficient to achieve identification of the model remaining parameters, since the order condition is still not satisfied. Identification is obtained by imposing the diagonality of $\mathbb{V}[\eta_t]$. Such a choice, which is economically motivated, allows satisfying the standard order condition for identification.

Nevertheless, further constraints are generally required on the model parameters. Starting from the spatial econometrics literature, that takes a scalar time invariant ρ coefficient and a time invariant row-normalized W matrix, we must impose that $\frac{1}{\lambda_{max}} < \rho < \frac{1}{\lambda_{min}}$ where λ_{max} and λ_{min} are, respectively, the maximum and minimum eigenvalues of W . This constraint ensures the non-singularity of $I - \rho W$.

In our framework we deviate from traditional approaches in several ways. We first consider the case of a time-varying spatial matrix, that is W_t . A sufficient condition for the invertibility of $I - \rho W : t$ for all t is stated in the following assumption

Assumption 4.1. *The coefficient ρ satisfies the following condition*

$$\bar{\lambda}_{max}^{-1} < \rho < \bar{\lambda}_{min}^{-1} \quad (41)$$

where

$$\bar{\lambda}_{max} = \min \{ \lambda_{t,max} \}_{t=1}^T \quad (42)$$

$$\bar{\lambda}_{min} = \max \{\lambda_{t,min}\}_{t=1}^T \quad (43)$$

and $\lambda_{t,max}$ and $\lambda_{t,min}$ are, respectively, the minimum and maximum eigenvalues of a matrix W_t . ■

If we have a diagonal matrix \mathcal{R} containing the asset-specific reaction to the spatial links, we assume the non-singularity which is then validated in the estimation step of the model:

Assumption 4.2. *The diagonal coefficient matrix \mathcal{R} is such that*

$$I - \mathcal{R}W_t \quad (44)$$

is non-singular for each matrix W_t . ■

Note that the previous assumption covers both the case of a time-invariant and time-varying spatial matrix. We further note that, when we consider a model with \mathcal{R} , we must impose an additional identification condition

Assumption 4.3. *The diagonal coefficient matrix $\mathcal{R} = \text{diag}(\rho_1, \rho_2, \dots, \rho_K)$ is such that $\rho_j = 0$ if the j -th row of the matrices W_t contains only zeros (that is the asset j is not linked to any other asset) for all values of t .* ■

This condition ensures that the asset specific impact to the network links is estimated only if such link exist for at least one point in time.

The use of covariance restrictions has a consequence for the estimation of model parameters. In fact, those must be jointly evaluated, despite the linear model structure might allow for single equation (single asset) parameter estimation.

Under the two strong parametric restrictions we impose (the structure on A and the absence of correlation across the idiosyncratic shocks), a viable approach is that of Full Information Maximum Likelihood (FIML) methods. However, if K is even moderately large, the total number of parameters to be estimated in the restricted structural model,

$7K$, might be quite large. Fortunately, we can follow the approaches commonly used in spatial econometrics, namely the use of concentrated likelihoods. As in Elhorst (2003), and LeSage and Pace (2009), we start by writing the full model log-likelihood

$$L(\Theta) = \sum_{j=1}^T l_t(\Theta), \quad (45)$$

$$l_t(\Theta) \propto -\frac{1}{2} \log|\Omega| - \frac{1}{2} e_t' \Omega^{-1} e_t, \quad (46)$$

$$e_t = R_t - \bar{\alpha} - \mathcal{R}W R_t - \bar{\beta} F_t. \quad (47)$$

where Ω is a diagonal matrix. We can note that, if ρ is known, we can write

$$R_t - \mathcal{R}W R_t = Z_t = \bar{\alpha} + \bar{\beta} F_t + \varepsilon_t \quad (48)$$

Therefore, with a known network exposure parameter matrix \mathcal{R} , we might estimate the parameters in $\bar{\alpha}$ and in $\bar{\beta}$ by least square methods, obtaining the well-known expressions. In addition, we might even recover standard estimators for the innovation variance. This suggests that the network exposure parameters can be easily obtained by maximizing the concentrated likelihood obtained by replacing the other parameters by their least square estimators.

This will be of a relevant computational importance as it allows reducing the parameters to be jointly estimated to $2K$ if we concentrate the likelihood with respect to $\bar{\alpha}$ and $\bar{\beta}$, and to K if we concentrate also with respect to the innovation variance. Standard errors can be recovered from the full-model likelihood by numerical evaluations of the Hessian (and of the gradient if we take a robust parameters covariance matrix). Note that this approach can be followed even if the spatial matrix W is time-varying.

5 Empirical analysis

PRELIMINARY AND INCOMPLETE

5.1 Data description

The empirical example we propose to highlight the features of the methodology we outlined above is based on the 48 US industry sectors portfolio returns available at the Kenneth French website. We fit the Carhart (1997) four factor model the model, and we recover the market, size, book-to-market and momentum factor from the same source, the Kenneth French website. We point the reader to the works of Fama and French (1995) and Carhart (1997), for the methodology behind the construction of the risk factors. We used both monthly and daily data: with the daily data we estimate the spatial matrix W as we will discuss below, while with the monthly data we estimate the model parameters, that is, the structural parameters associated with the risk factors and the sector-specific spatial parameters ρ_i included in the matrix \mathcal{R} .

The sample period we consider starts in January 2000 and ends in December 2013.

[To be included: descriptive statistics - we must decide if we want to include the range 1993-1999, by now it is not included]

5.2 A benchmark model: the 4-factor CAPM

We first estimate on our data and on both subsamples a reference model, the four-factor CAPM of Carhart (1997). We provide in Table (1) the point values of estimated betas for selected economic sectors: three financial sectors, Banks, Insurances and Real Estate, and three industrial sectors, Autos, Machinery and Chemicals. The selection is clearly arbitrary but is also motivated by the relevance of the financial side of the US economy in the subprime crises and by the impact the crisis had on the real economy, represented by the industrial sectors. The appendix reports the results for the full list of economic

sectors.¹²

Table 1: 4-factor betas for selected economic sectors

Sectors/Betas	Market	SMB	HML	Momentum
2000-2006				
Chems	1.04*	-0.17	0.57*	0.25*
Mach	1.35*	0.40*	0.41*	0.08
Autos	1.22*	0.41*	1.14*	0.20
Banks	0.89*	-0.30*	0.58*	0.17*
Insur	0.88*	-0.49*	0.63*	0.36*
REEst	0.55*	0.38*	0.44*	0.21*
2007-2012				
Chems	1.31*	0.13	-0.30	-0.21
Mach	1.49*	0.41*	-0.40*	-0.16
Autos	1.34*	0.78*	0.29	-0.25
Banks	0.89*	-0.34*	1.64*	0.39*
Insur	1.13*	-0.23	0.40*	-0.01
REEst	1.23*	1.34*	0.72*	-0.14

(*) denotes 5% significant coefficients

The betas with respect to the market index are all statistically significant and positive, as expected. By comparing the two sub-samples we point out a surprising findings, associated with the Banks sector. For that sector the beta is stable in the two periods, while for all other sectors we note an increase in the exposure to the market.

For the size factor (SMB) the estimates are quite heterogeneous across the six sectors, but we highlight the large increase of the Real Estate sector reaction to the size factor. Moving to the book-to-price factor, we have a general decrease of the betas from 2000-2006 to 2007-2013. In two cases, for the Chemicals and Machinery sectors, the beta becomes negative while still being statistically significant. Similarly to the market case, the Banks sector moves at odds with the remaining ones showing an increase in the exposure. The last finding is also present in the betas associated with the Momentum factor that decrease and become negative for all sectors Banks excluded where the beta increases and is statistically significant in the second sub-sample (it is the only statistically significant beta in that period).

¹²Appendix available upon request.

5.3 Network Estimation

Network interconnections could be determined by just looking at balance sheet direct exposures (for instance when dealing with financial companies) or estimated from market data. There are several methods proposed in the financial literature, as examples of contributions on network analysis we mention, among others: Billio et al. (2012) that propose a Granger causality approach to detect network connections; the Diebold and Yilmaz (2013) approach is based on variance decompositions; Hautsch, Schaumburg, and Schienle (2012, 2013) adopt a two-stage quantile regression approach to determine the firms' tail risk exposures; Barigozzi and Brownlees (2013) suggest the estimation of cross-sectional conditional dependence to represent network interconnections.

Our methodology could be implemented using any of these approaches. In fact, the model takes the network as an input of the analysis, and is thus conditional to an estimated network. In the following empirical example we use an extension of the Granger causality method proposed by Billio et al. (2012).

We also stress that we estimate the spatial matrix for each year using the daily data. As a consequence, we allow for a time-change in the network structure, inducing thus a time-variation in the contemporaneous coefficient matrix A_t . We stress that this choice induces a mild heteroskedasticity in the reduced form model. As in Billio et al (2012), we use a GARCH(1,1) to filter out from the daily returns the known heteroscedasticity. Given the log return series

$$r_{i,t} = \mu_i + \eta_{i,t} \quad (49)$$

where μ_i is the conditional mean and $\eta_{i,t}$ is the innovation for asset i . Following the standard literature, we set $\eta_{it} = \sigma_{it}\epsilon_{it}$ where σ_t is the conditional standard deviation. The conditional variance follows a simple GARCH(1,1) process

$$\sigma_{it}^2 = \omega_i + \alpha_i \eta_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \quad (50)$$

with $\omega_i \geq 0$, $\alpha_{i,1} \geq 0$, $\beta_i \geq 0$, and $\alpha_i + \beta_i < 1$. In the GARCH literature $\epsilon_{i,t}$ is assumed to

be a sequence of i.i.d random variables with zero mean and unit variance. Therefore, under this assumption, we are able to compute the so-called standardized residuals $\epsilon_{it} = \frac{\eta_{it}}{\sigma_{it}}$. On the standardized residuals we then apply the Granger Causality test on a bi-variate basis, that is considering pairs of economic sectors. From an analytical viewpoint, we consider the following equation:

$$\epsilon_{i,t} = \sum_{k=1}^m a_k \epsilon_{i,t-k} + \sum_{k=1}^m b_k \epsilon_{j,t-k} + \sum_{k=1}^m c_k \epsilon_{l,t-k} + \sum_{k=1}^m d_k F_{t-k} + \varrho_{i,t} \quad (51)$$

where $\epsilon_{i,t}$ is the standardised residual of asset i , the dependent variable, a_k b_k c_k d_k are the model coefficients, while $\varrho_{i,t}$ is the uncorrelated white noises and k is the lag. The model includes several *explanatory* variables: the “causing series” $\epsilon_{j,t}$ and the lagged dependent variable, $\epsilon_{i,t-k}$, are standard ingredients in the VAR model behind the Granger causality test; we added here a third series, called the “background series” $\epsilon_{l,t}$ and a common factor F_t . The last two addition to the baseline VAR model are included in order to make robustify the Granger causality test; we stress that, the bivariate model where we run the Granger causality test is thus a VARX. The background series $\epsilon_{l,t}$ is taken from the set of remaining 46 sectors time series, so that $l \neq i, j$. In addition, we set the common factor F_t to be the market portfolio. In order to determine which is the preferred model specification, for each pair of causing-caused series $\epsilon_{i,t}$ and $\epsilon_{j,t}$ we run a Granger Causality test for each possible background series $\epsilon_{m,t}$. Thus, for each pair of causing-caused series we have 46 different test statistics determining if $\epsilon_{j,t}$ causes in sense of Granger $\epsilon_{i,t}$. The causality presence is associated with the coefficients a_k being statistically different from zero.

Among the 46 test statistics, we place ourselves on the safe side by picking the worst case, that is we choose the regression having the higher p-value on the causality tests. We will thus detect the presence of causality by taking into account a large number of system specification. The causality presence, as detected with the above outlined procedure, is our reference to determine the spatial matrix and the associated network structure. In fact we compute the adjacency matrix (spatial matrix) by setting $w_{i,j} = 1$ when the p-

value of the test on the significance of parameters a_k of the reference regression for asset i suggests that asset j Granger-cause asset i at the 5% confidence level.

For each year, we estimate the spatial Matrix W_t and we also row-normalize the matrices in order to equalized the impact of each unit on all other units, using a standard procedure in Spatial econometrics, see Elhroost(2003).

To fully appreciate the impact of Granger-causal relationships among various industries, we provide a visualization of the results of linear Granger-causality tests, applied over the 14 years sample used.

Granger-causality relationships are drawn as straight lines connecting two economic sectors, color-coded by the type of sector that is “causing” the relationship, i.e., the sector at date- $t - 1$ which Granger-causes the returns of another sector at date t . Only those relationships significant at the 5% level are depicted. For reasons of space, we report plots only for two of the 21 yearly networks in Figures 10 and 11: 2002 and 2008. These are representative time periods encompassing both tranquil and crisis periods in the sample.¹³ We see that the number of connections between different sectors dramatically increases from 2002 to 2008.

The graph’s density is 7% for the year 2002 and 15.9% for the year 2008. The density is given by the the ratio $g = \frac{\bar{N}}{N}$ where \bar{N} is the number of link among the assets and N is all the possible links among the assets.

5.4 Results

We then move to the estimation of the factor model augmented with the contemporaneous links across economic sectors. We stress that, while the networks are estimated on the basis of daily data, and with yearly update of the network, the return model is fit on monthly data.

The sample period we consider ranges, as before, from 2000 until 2013. Since we expect that not only connections change through time but also the stock return exposures

¹³To fully appreciate the dynamic nature of these connections, see the Appendix, available upon request.

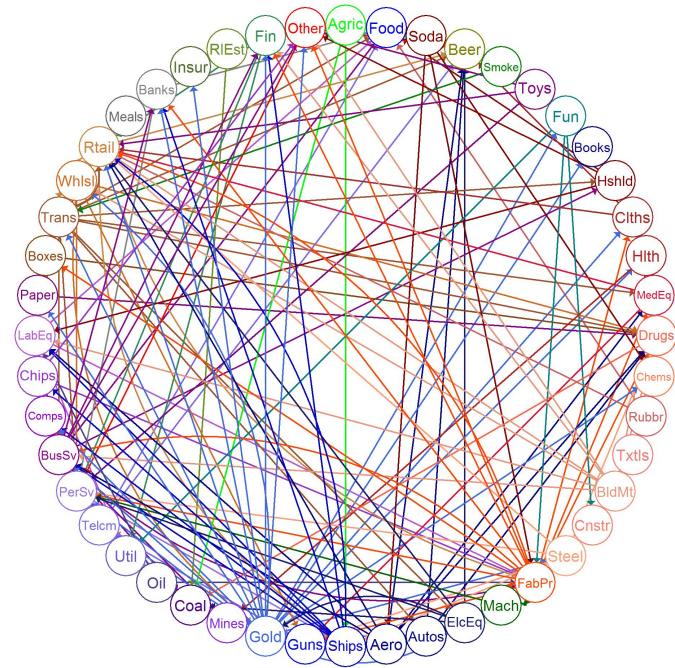


Figure 10: Estimated Network for the year 2002

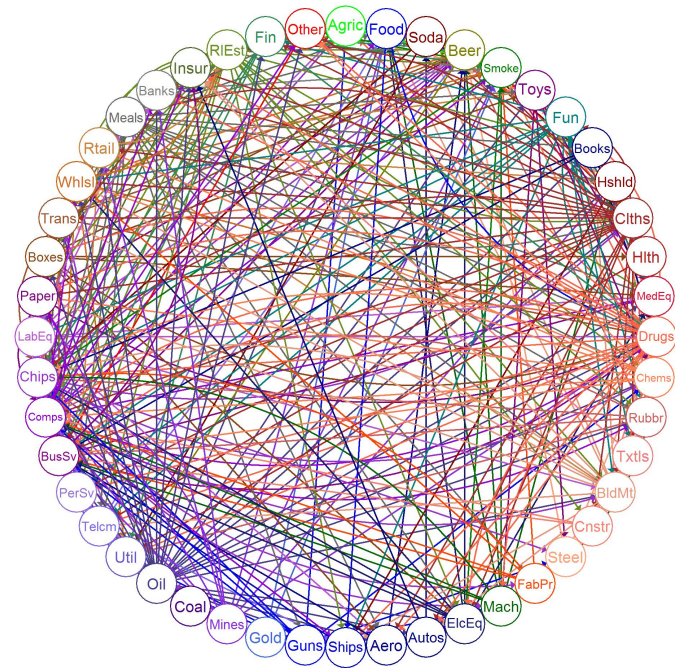


Figure 11: Estimated Network for the year 2008

to the network connections changes over time, in particular as a consequence of the global financial crisis of 2007-2009, we split the sample into two intervals. We thus provide two different estimates of the ρ vector (the diagonal parameter vector of the matrix \mathcal{R}): ρ_{00-06} for the period 2000 – 2006 and ρ_{07-13} for the period 2007 – 2013. Each vector collects the spatial impact coming from the neighbors on the sectors for that period of time. As we used monthly returns and yearly spatial matrices, we are able to compare for the same industrial sector both its spatial impact for different time periods and its risk loading deriving from common factor and from the network exposure. The parameters are estimated as described in section 4.

Figure 12) shows the estimated spatial parameters ρ vectors for 6 of the 48 industrial sectors available from the data sample (the same used in Section 5.2) and shows that during the first period 2000-2006 the spatial parameters ρ vectors associated with financial institutions (banks and insurances) were lower than the second period of time 2007-2013. In contrast, for the non financial sectors (Auto, Chemical, Machinery) the spatial parameters values reduce from the first interval to the second temporal interval.

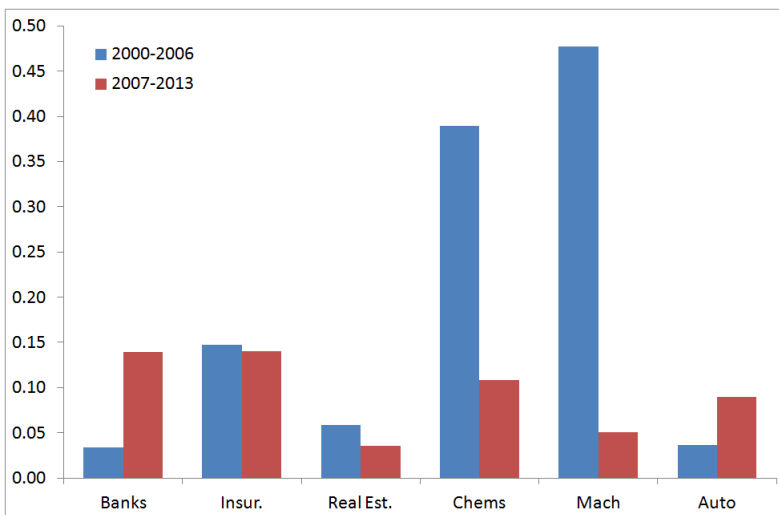


Figure 12: Estimated spatial parameters ρ for the period 2000 – 2006 and for the period 2007 – 2013

The Appendix contains the full set of estimated ρ_i coefficients while selected rhos are included in Table (2).¹⁴ Figure (13) report a comparison of the spatial parameters ρ_i

¹⁴Appendix available upon request.

across the economic sectors and the two sample periods. We first note that the points are clustered around the zeros, thus suggesting that the links across assets has a limited (despite statistically significant) impact. Moreover, there is not a clear pattern between the two periods with network impacts that can be either increase (points above the dashed line) or decrease (below the dashed line) from 2000-2006 to 2007-2013. Such an heterogeneous behavior might correspond to a different impact depending on the economic sector reaction to the crises. In fact, as we previously noted, financial sectors and industrial sectors show a different movement in the ρ coefficients. Furthermore, we note that some coefficient are negative, either in one of the two periods (more frequently in the second subsample) or in both periods. Such a finding is coherent with the presence of negative correlations across the multifactor model residuals and suggests the presence of a risk absorption in some cases. This might be also interpreted as a sort of disintegration across sectors, with opposite reactions during and after the crises.

Table 2: Spatial Parameters 6 of 48 sectors for 2000-2006

Sectors/Period	2000-2006	2007-2013
Chems	0.39*	0.11*
Mach	0.48*	0.05*
Autos	0.04*	0.09*
Banks	0.03*	0.14*
Insur	0.15*	0.14*
RIEst	0.06*	0.04*

(*) denotes 5% significant coefficients

Table (3) report the structural betas for selected sectors (see the Appendix, available upon request, for tables with results on all sectors). We first note that the estimated betas are not much different from those of the traditional model. Some slight differences are present, in particular for the significance of the Fama-French and Carhart risk factors.

However, one of the advantages of our model, is its ability to combine a structural behavior with an impact coming from the network exposure. Building on this aspect, the model is able to separate for each industrial sector the exposure to systematic risk and the exposure to risk coming from the spatial interactions. Therefore the beta of the

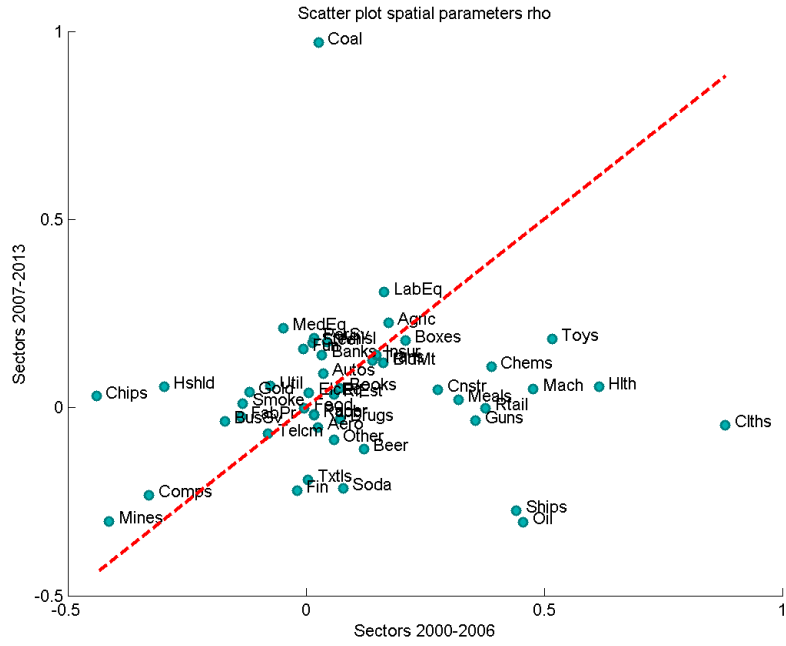


Figure 13: Estimated spatial parameters ρ for the period 2000 – 2006 versus those of period 2007 – 2013

Table 3: Structural betas for selected economic sectors

Sectors/Betas	Market	SMB	HML	Momentum
2000-2006				
Chems	0.83*	-0.07	0.43	0.18
Mach	0.95*	0.24*	0.18	-0.04
Autos	1.19*	0.41	1.13*	0.19
Banks	0.86*	-0.30	0.55	0.16
Insur	0.79*	-0.50	0.55	0.31
RIEst	0.51*	0.37*	0.41	0.19
2007-2012				
Chems	1.23*	0.14	-0.38	-0.22
Mach	1.46*	0.41*	-0.39	-0.16
Autos	1.28*	0.78	0.32	-0.24
Banks	0.83*	-0.33*	1.63*	0.44*
Insur	1.04*	-0.25	0.40*	0.06
RIEst	1.20*	1.36*	0.72*	-0.12

(*) denotes 5% significant coefficients

reduced form model, $\bar{\beta}$, can be split into two components: the exposure to the common factors and the exposure to the network. This has been shown in equation (24) with a constant spatial matrix W . However, with a smoothly evolving matrix we have the following decomposition:

$$\begin{aligned}\bar{\beta}_t^* &= \bar{\beta} + \sum_{j=1}^{\infty} \mathcal{R}^j W_t^j \bar{\beta} & (52) \\ &= \bar{\beta} + \bar{\beta}_t^{Net} & (53)\end{aligned}$$

where $\bar{\beta}_t^*$ is the time-varying reduced form beta matrix. The latter is given as the sum of a time-invariant, structural, term $\bar{\beta}$ and a time-varying component $\bar{\beta}_t^{Net}$ influenced by the time-evolution of the spatial matrix W_t . While the structural component is directly obtained from the model estimation, the reduced form betas can be computed only conditionally to the estimated model coefficients. Similarly to the case of ρ parameters, we obtain two different structural beta matrices referring, respectively, to the two samples: $\bar{\beta}_{0-06}$ from January 2000 to December 2006 and $\bar{\beta}_{07-13}$ for January 2007 to December 2013. Each $\bar{\beta}$ matrix contains the exposure to the four risk factors we consider.

The fact that the reduced-form betas are time-varying implies that the impact of network connections is time-varying. Therefore, while the sign of the network impact is only driven by the \mathcal{R} matrix, the size of the network impact depends both on the parameters and the network structure implicit in W_t . With positive ρ_j coefficients, the inflation in the factor exposures might be further amplified by the network structure.

We first provide some insight from the graphical representation of the average network impact on the betas. We thus average over the yearly reduced form betas as follows:

$$\bar{\beta}_{00-06}^* = \frac{1}{7} \sum_{j=2000}^{2007} \bar{\beta}_j^* \quad (54)$$

$$= \bar{\beta} + \sum_{j=2000}^{2007} \bar{\beta}_j^{Net} \quad (55)$$

and similarly for the second subsample.

Figure (14) reveals the average exposure to the market risk for 6 out of the 48 sectors in the two samples. It is worth noticing that for all sectors the overall exposure to the market risk increases from the first to the second period. However, the change is due to a pure increase in the structural exposure for the real economy sectors, while for financial sectors, and in particular for the banking sector, the change is driven by a modification in the network exposure.

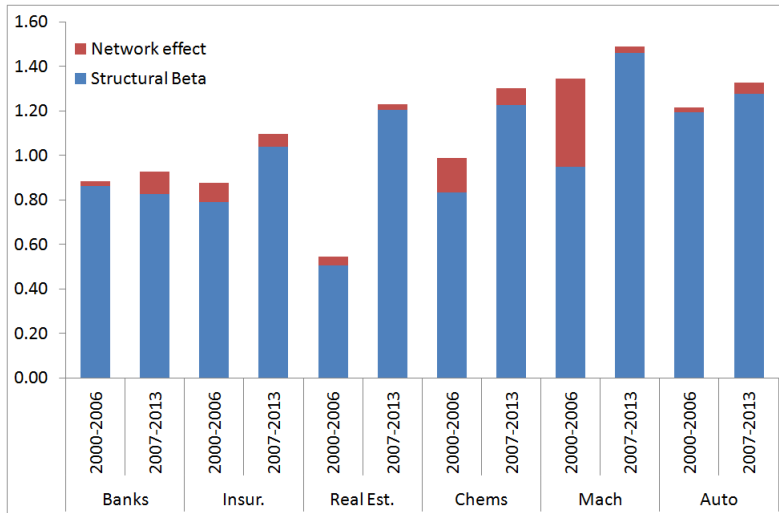


Figure 14: The graph captures for each period 2000 – 2006 and 2007 – 2013 the exposure to risk, distinguishing the systematic contribution and the network contribution

To provide further insights we plot the reduced form yearly betas. We remind that, by construction, the reduced form betas are time-varying, thanks to the time change of the spatial matrices W_t .

The previous analyses focus on the estimated parameters and on their variation across economic sectors and over time. We now move to a different framework and evaluate the

impact of both the systemic and network exposure on the portfolio risk. We note that, according to equation (35) the total risk is given as a sum of four components. Given the estimation strategy adopted, the first and third components, that is the structural systematic and structural idiosyncratic ones, are constant on the two sub-samples 2000-2006 and 2007-2013. On the contrary, the second and fourth components, the network impacts on the systematic and idiosyncratic risk, are time-varying and change on a yearly basis as they depend on the spatial link matrices W_t . We perform thus perform a first evaluation at two specific points in time, 2006 and 2013. We consider an equally weighted allocation strategy and vary the portfolio size (in terms of assets number), starting from a 5-sector portfolio up to a 48-sector portfolio. Since the ordering of economic sectors might have an impact, we select two possible rankings: we order sectors by their total risk or by the impact of the network exposure (the ρ_j coefficients). In both cases the ordering is from the lower to the higher value of the two indicators. Then, for each portfolio we report in Figures from (15) to (18) the absolute contribution to the portfolio total risk of the four components of (35).

Some elements clearly emerge. Firstly, the systematic component has a predominant role independently from the sector ordering and the sample. Secondly, the network impact on the idiosyncratic risk is almost irrelevant, and the idiosyncratic component has a minor role. The latter finding might be seen as a confirmation of the appropriateness of the four-factor model (augmented with the network exposure) in capturing the risk sources affecting the economic sectors. Thirdly, we note that the risk absorption has a relevant role if we order assets on the basis of the ρ_j coefficients, a somewhat expected finding. Fourthly, the network exposure on the systematic risk is more clear in the first subsample and the total risk sensibly increases from 2006 to 2013. This might suggest that the financial crisis had an effect mostly on the structural systematic exposure to the risk factors while the network impact role is decreased.

A closer look at this last element might come from a yearly evaluation of the variance decomposition, see Table (4). We remind that the first and third components (the

structural systematic and idiosyncratic terms) are constant over sub-samples. On the contrary, the risk contribution from the network impact, either on the systematic or the idiosyncratic terms, is time-varying. We note that the overall risk is sensibly higher in the second subsample due to the increase in the systematic component. This signals the relevant impact of the crisis that has modified on the one side the risk of the common factors and on the other side the reaction to the common factors. Secondly, we observe that the idiosyncratic risk contributions have a minor impact, less than 5% from 2000 to 2006 and even smaller from 2007 to 2013. We read this as both a by-product of the diversification, the ability of the structural model to capture the common risk exposures, and, for the decrease in the second sub-sample, as a consequence of the large increase in the systematic risks. The links across economic sectors have some impact on the systematic risks, much higher in the first subsample, in general higher than 10%. The decreased relevance from 2007-2013 is, in our opinion, again a consequence of the increase in the structural risk exposure to systematic risks and can be read as an effect of the global diffusion of the crisis.

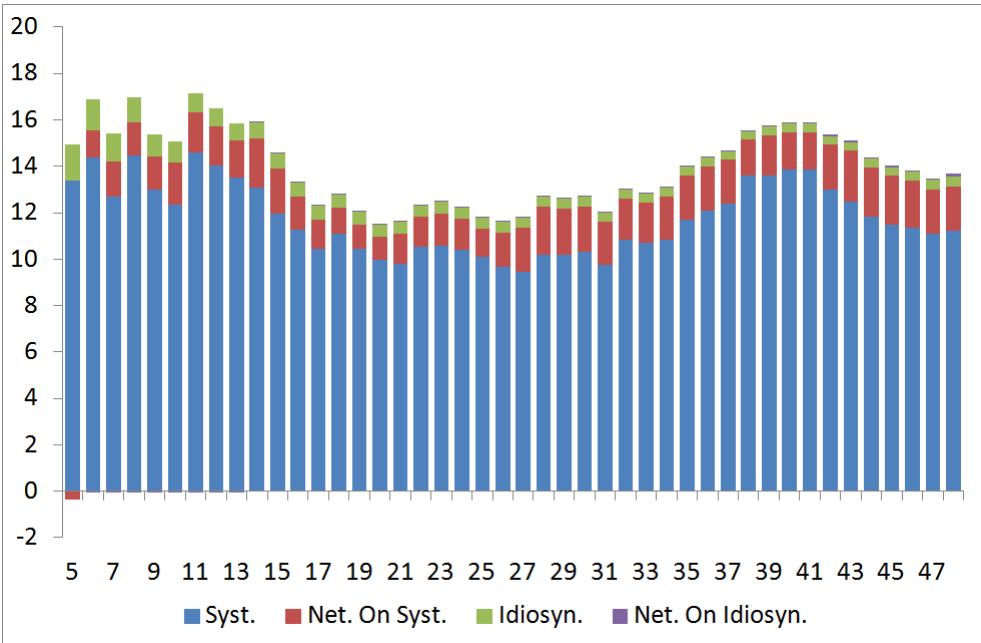


Figure 15: Variance decomposition for equally weighted portfolio with different number of assets; 2006 decomposition with assets ordered by increasing total risk.

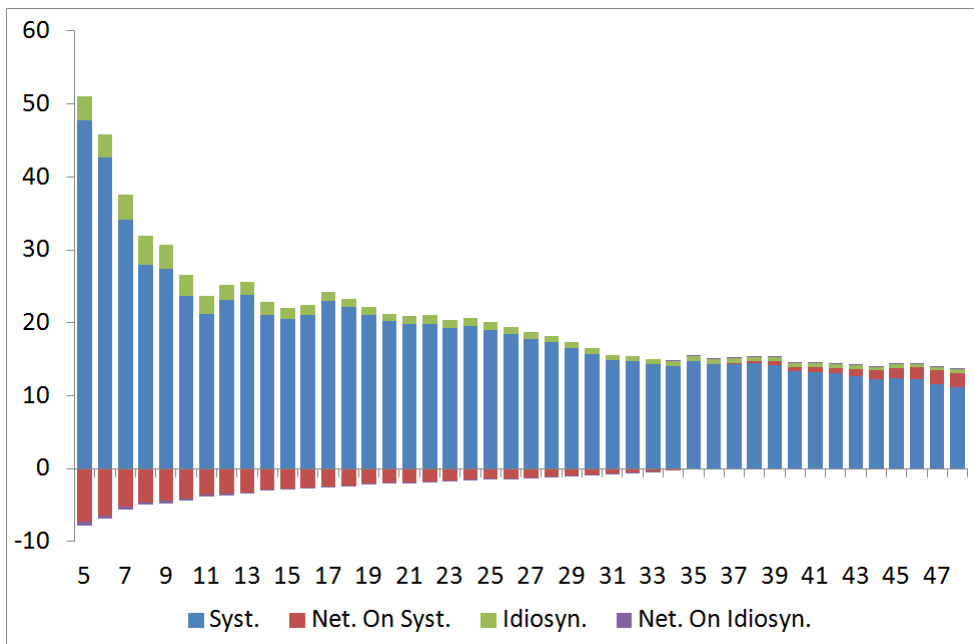


Figure 16: Variance decomposition for equally weighted portfolio with different number of assets; 2006 decomposition with assets ordered by increasing ρ_j coefficient.

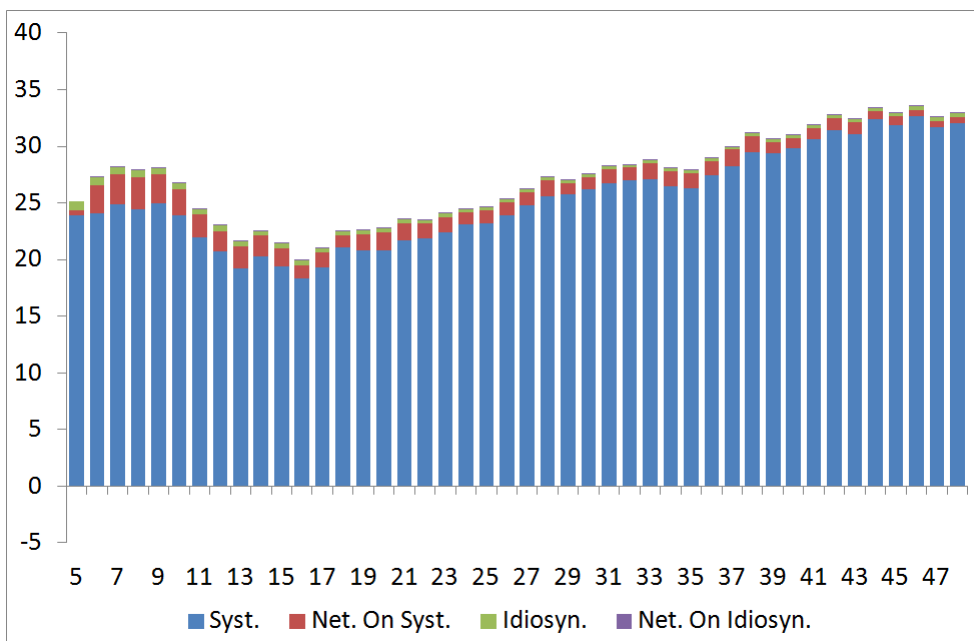


Figure 17: Variance decomposition for equally weighted portfolio with different number of assets; 2013 decomposition with assets ordered by increasing total risk.

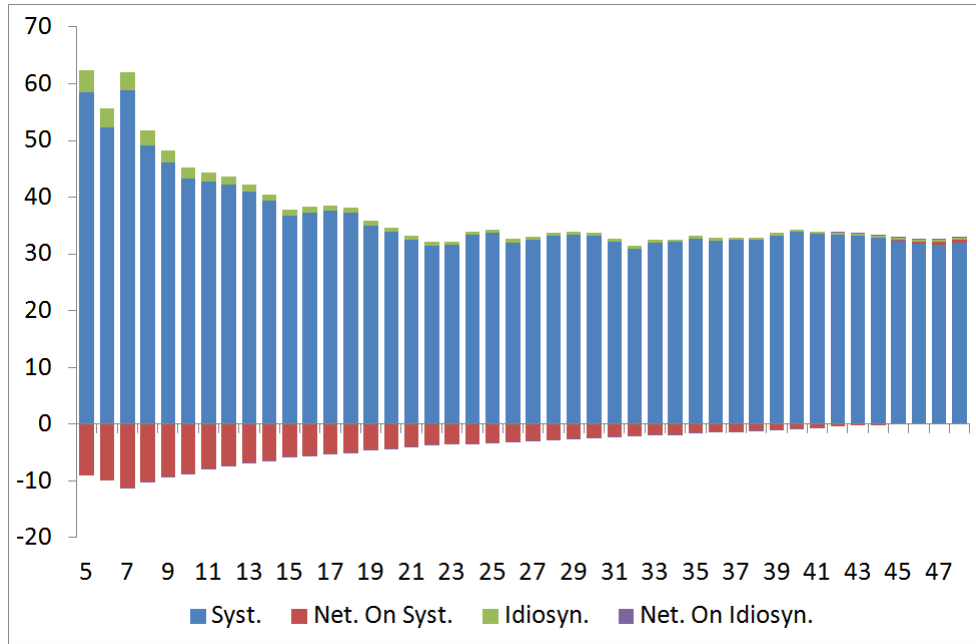


Figure 18: Variance decomposition for equally weighted portfolio with different number of assets; 2013 decomposition with assets ordered by increasing ρ_j coefficient.

Table 4: Variance decomposition

Year	Absolute				Relative			
	I	II	III	IV	I	II	III	IV
2000	11.233	2.205	0.454	0.097	80.30%	15.76%	3.24%	0.69%
2001	11.233	2.651	0.454	0.140	77.59%	18.31%	3.14%	0.97%
2002	11.233	2.001	0.454	0.131	81.29%	14.48%	3.28%	0.95%
2003	11.233	2.038	0.454	0.191	80.72%	14.65%	3.26%	1.37%
2004	11.233	0.524	0.454	0.076	91.42%	4.26%	3.69%	0.62%
2005	11.233	1.720	0.454	0.078	83.30%	12.76%	3.37%	0.58%
2006	11.233	1.890	0.454	0.108	82.08%	13.81%	3.32%	0.79%
2007	32.050	1.744	0.362	0.023	93.77%	5.10%	1.06%	0.07%
2008	32.050	0.580	0.362	0.015	97.10%	1.76%	1.10%	0.05%
2009	32.050	0.577	0.362	0.005	97.14%	1.75%	1.10%	0.01%
2010	32.050	0.726	0.362	0.007	96.70%	2.19%	1.09%	0.02%
2011	32.050	4.270	0.362	0.114	87.10%	11.60%	0.98%	0.31%
2012	32.050	0.713	0.362	0.007	96.73%	2.15%	1.09%	0.02%
2013	32.050	0.505	0.362	0.008	97.34%	1.53%	1.10%	0.02%

The table reports the decomposition of the variance for the equally weighted portfolio composed by the 48 economic sectors. The components reflects the contribution of the systematic structural risk (I - constant across subsamples), idiosyncratic structural risk (III - constant across subsamples), network impact on systematic risk (II) and network impact on idiosyncratic risk (IV).

6 Conclusions

TO BE INCLUDED

References

- [1] Abreu, M., de Groot, H.L.F., and Florax, R.J.G.M., 2005, Space and growth: a survey of empirical evidence and methods, *Region et Developpement*, 21, 13-44.
- [2] Anselin, L., 1988, *Spatial Econometrics: methods and models*, Kluwer Academic Publisher.
- [3] Anselin, L., Florax, R., and Rey, S.J., 2004, *Advances in spatial econometrics: methodology, tools and applications*, Springer-Verlag, Berlin.
- [4] Arnold, M., Stahlberg, S., and Wied, D., 2013, Modeling different kinds of spatial dependence in stock returns, *Empirical Economics*, 44, 761-774.
- [5] Asgharian, H., Hess, W., and Liu, L., 2013, A spatial analysis of international stock market linkages, *Journal of Banking and Finance*, 37, 4738-4754.
- [6] Blasques, F., Koopman, S.J., Lucas, A., and Schaumburg, J., 2014, Spillover Dynamics for Systemic Risk Measurement Using Spatial Financial Time Series Models, SYRTO Working Paper 16.
- [7] Caporin, M., and Paruolo, P., 2013, Proximity-structured multivariate volatility models, *Econometrics Reviews*, forthcoming.
- [8] Carhart, M., 1997, On the persistence of mutual fund performance, *Journal of Finance*, 52, 1-133.
- [9] Conley, T.G. and Topa, G., 2002, Socio-economic Distance and Spatial Patterns in Unemployment, *Journal of Applied Econometrics*, 17, pp. 303-327.

- [10] Elhorst, J.P., 2003, Specification and estimation of spatial panel data models, *International Regional Science Review*, 26, 244-268.
- [11] Fama, E., and French, K.R., 1993, Common risk factors in the returns of stocks and bonds, *Journal of Financial Economics*, 33, 3-53.
- [12] Fama, E., and French, K.R., 1995, Size and book-to-market factors in earnings and returns, *Journal of Finance*, 50-1, 131-155.
- [13] Fernandez, V., 2011, Spatial linkages in international financial markets, *Quantitative Finance*, 11, 237-245.
- [14] Fernandez-Aviles, G., Montero, Jose-Maria, and Orlov, A.G., 2012, Spatial modeling of stock market comovements, *Finance Research Letters*, 9, 202-212.
- [15] Holloway, G., Lacombe, D., and LeSage, J.P., 2007, Spatial Econometric Issues for Bio-Economic and Land-Use Modelling, *Journal of Agricultural Economics*, 58, 549-588.
- [16] Keiler, S., and Eder, A., 2013, CDS spreads and systemic risk, a spatial econometric approach, *Deutsche Bundesbank Discussion Paper*, No. 1/2013.
- [17] LeSage, J. and Pace, R.K., 2008, Spatial Econometric Modeling of Origin-Destination Flows, *Journal of Regional Science*, 48, pp. 941-967.
- [18] LeSage, J. and Pace, R.K., 2009, *Introduction to spatial econometrics*, CRC Press.
- [19] Lintner, J., 1965a, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics*, 47, 13-37.
- [20] Lintner, J., 1965b, Security prices, risk and maximal gains from diversification, *Journal of Finance*, 20, 587-615.
- [21] Markowitz, H., 1952, Portfolio selection, *Journal of Finance*, 7, 77-91

- [22] Markowitz, H., 1959, Portfolio selection: efficient diversification of investments, Cowles Foundation Monographs, 16, New York, John Wiley & Sons.
- [23] Mossin, J., 1966, Equilibrium in a capital asset market, *Econometrica*, 35, 768-783.
- [24] Ross, S.A., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory*, 13, 341-360.
- [25] Sharpe, W.F., 1964, Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, *Journal of Finance*, 19, 425-442.
- [26] Wied, D., 2013, CUSUM-type testing for changing parameters in a spatial autoregressive model for stock returns, *Journal of Time Series Analysis*, 34, 221-229